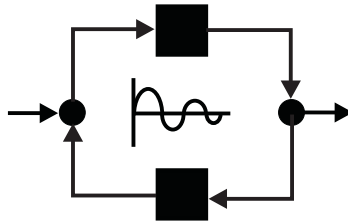




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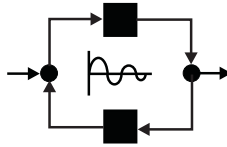
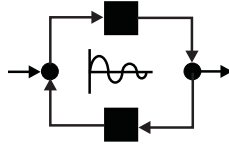


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Optimal T-S models for identification of nonlinear systems from input-output data

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Abstract

Determining optimal structure identification of a nonlinear black box system is one of the most significant steps in Fuzzy modelling based on Takagi-Sugeno models. The one parameter that needs to be determined before performing fuzzy clustering and identification algorithms is the optimal number of clusters. In this paper, we present a new approach for automatically predicting the optimal choice of this parameter simply by detecting the general trend of the data structure. First, an approximation model of the system is built by fitting data to a polynomial function. Second, a preliminary decomposition of the data is realized based on detection of the function turning points. Then, a merging method is adopted to reduce the identified number of clusters. The advantage of the generated solution is that it remains in the horizon of the data; hence there is no need to apply heuristic rules or conventional validation tools. The performance of the proposed method is evaluated for both quality of clustering and fuzzy modeling with 1st order TS systems using GK fuzzy algorithm.

Keywords: *Fuzzy clustering, optimal clusters number, nonlinear system, polynomial regression, Takagi-Sugeno models, GK algorithm.*

1 Introduction

Fuzzy clustering combining with regression analysis are the most tools used to capture sub linear systems in a structure of data, particularly for modelling an unknown black box nonlinear system with Takagi-Sugeno models [3]. There are two main issues in the process of constructing a TS fuzzy model, the first is how to determine the number of rules and the variables involved in the rule antecedents, and the second is how to estimate the parameters of the TS fuzzy model. To construct TS fuzzy model from data, many fuzzy clustering algorithms combined with least

square method have proved to be suitable techniques to partition data space into adequate subspaces and detect linear local models[5].

The Fuzzy C-Means(FCM) and the Gustafson-Kessel(GK) fuzzy clustering algorithms are the most widely used to partition a data set into c homogeneous fuzzy clusters. The FCM detects clusters of a roughly same size. The GK algorithm is an extension of the FCM, which can detect clusters of different orientation and shape in a data set. Both the FCM and the GK algorithm require the number of clusters as an input, and the analysis result can vary greatly depending on the value chosen for this variable. However, in many cases the exact number of clusters in a data set is not known. Both the FCM and the GK algorithm may lead to undesired results if a wrong cluster number is given. The number of clusters needs to be known a priori and the correct number of rules must be estimated in advance[1, 10].

To overcome this problem, many approaches have been proposed: in the first framework, modelling procedure constitutes an iterative process, where the number of rules gradually increases until the model's performance meets a predefined accuracy[7, 12], the second modelling framework is based on using optimal fuzzy clustering, starting with a sufficiently large number of clusters and successively reducing this number by merging clusters that are similar (compatible) with respect to some predefined criteria [11, 13].

In this work we present a new technique to determine the optimal number of clusters, T-S models, that should be used for modelling a given data set of a nonlinear black box system. The basic idea is to fit a polynomial function which follows the general trend in data. The global structure of data space is then recognized by the characteristics points: maxima and minima. Each group of data ranging between maximum and minima, considered a cluster, can be approximated by a linear model. So the total number of cluster, i.e. linear models, can be deduced from the number of turning points. To find the optimal



number, a cluster merging technique is proposed. It consists on fusion of consecutive clusters for which the associated linear model are almost Collinear. The performances of results are examined by using the so called validity clustering criteria and the quality of the approximation is evaluated by comparing fuzzy models obtained by the MATLAB Fuzzy Toolbox and the fuzzy Model identification based on Gustafson-Kessel clustering algorithm.

The basic advantage of the proposed method when compared to other works, is that the optimal number of TS models obtained may be used as an input parameter by any TS identification algorithm to generate an optimal fuzzy model without need to apply heuristic rules or conventional tools to validate the results. In more, the performance of the most indices found in the literature depends on the algorithm used and the structure of the studied system [6, 15].

This paper is organized as follows. Structure of TS fuzzy model is presented in section 2. Section 3 describes the method for construction of the approximation polynomial function, the partitioning method for extraction of the global number of clusters and the merging technique. Section 4 presents simulation results. Section 5 gives some concluding remarks and discuss current and future extensions of this work.

2 Takagi-Sugeno fuzzy models

Consider the identification of an unknown nonlinear Black Box system of the form :

$$y_k = f(x_k)$$

based on specified or measured input data X and measured output data Y of the system, where $k = 1, \dots, N$, denotes the index of the k -th input-output data-pair. In general it may not be easy to find a global nonlinear model that is universally applicable to describe the unknown system $f(\cdot)$. In that case it would certainly be worthwhile to build local linear models for specific region of the data space and combine these into a global model. The relation between input and the output of the process is then described by linguistic if-then rules with fuzzy proposition in the antecedent and crisp functions in the consequents. Hence, it can be seen as a combination of linguistic and mathematical regression modeling in the sense that the antecedents describe fuzzy regions in the input space in which consequent functions are valid. The TS rules have the following form::

$$R_i : \text{if } x \text{ is } A_i \text{ then } y_i = a_i x + b_i \quad i = 1, 2, \dots, c$$

where c is the number of if-then rules, A_i are fuzzy sets, and (a_i, b_i) are the model consequent parameters that have to be identified in a given data set. For a given input crisp vector $x = (x_1, x_2, \dots, x_N)^T$ the inferred global output of the Takagi-Sugeno model is

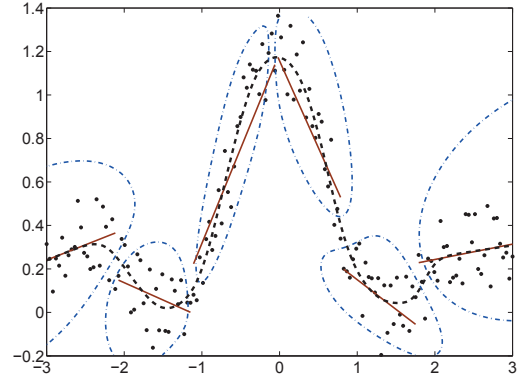


Figure 1: Results of the GK algorithm for fuzzy modelling of a black box nonlinear system : Fuzzy model, TS models and Clusters.

computed by taking the weighted average of the individual rules contributions

$$y = \frac{\sum_{i=1}^c \mu_i(x) \cdot y_i}{\sum_{i=1}^c \mu_i(x)}$$

where $\mu_i(x)$ is the degree of fulfillment of the i^{th} fuzzy rule.

The Gustafson-Kessel (GK) clustering algorithm has often been applied to identify automatically TS models. In general, fuzzy clustering algorithms generate a fuzzy partition given as a fuzzy partition matrix $U = [\mu_{ij}]$, where $\mu_{ij} = \mu_{\tilde{F}_i}(x_j)$ is the membership value of the data x_j belonging to the fuzzy cluster $\tilde{F}_i$. The objective of GK algorithm is to obtain a fuzzy c -partition $\tilde{F} = \{\tilde{F}_1, \dots, \tilde{F}_c\}$ for the given number of clusters c and the given N input-output data (X, Y) by minimizing the evaluation function J_m ,

$$J_m(U, V, A : X) = \sum_{i=1}^c \sum_{j=1}^N (\mu_{ij})^m D_{ijA_i}^2$$

$$D_{ijA_i}^2 = (x_j - v_i)^T A_i (x_j - v_i)$$

where $V = (v_1, \dots, v_c)$ is a vector of the centroids of the fuzzy clusters $(\tilde{F}_1, \dots, \tilde{F}_c)$, m controls the fuzziness of membership of each datum, D is an adaptive norm (Mahalanobis distance) used by the GK algorithm for each cluster to detect different geometrical shapes in data sets. Each i^{th} cluster has its own norm-inducing matrix A_i , where A_i is obtained from fuzzy covariance matrix (F_i) of the i^{th} cluster defined by:

$$F_i = \frac{\sum_{j=1}^N \mu_{ij}^m (x_j - v_i)(x_j - v_i)^T}{\sum_{j=1}^N \mu_{ij}^m}$$

$$A_i = [\rho_i \det(F_i)]^{-1/p} F_i^{-1}$$

with $\rho_i > 0 \quad \forall i$, p is the data space dimension. Without any prior knowledge, the cluster volume ρ_i is simply fixed at 1 for each cluster, which allows the algorithm to find clusters of approximately equal



volumes[2].

It should be noted that for the process of identification of T-S models based on GK fuzzy clustering algorithm, the number of rules coincides with the number of clusters, each cluster is approximated with a local linear model(Figure1).

3 Method formulation

In this section , we present our method in tree steps: identification of the polynomial model, extracting of the global number of clusters from the approximation function and the optimization of T-S models by the merging thechnique (Figure 2) .

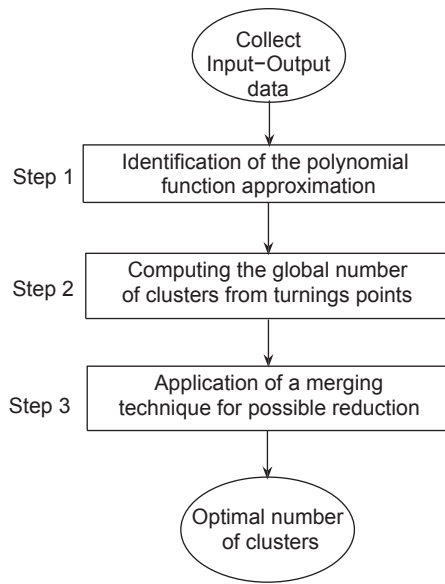


Figure 2: Process for identification of the optimal number of TS models.

3.1 Data model identification

The general problem studied in this section can be stated as follows. On the basis of N input-output samples

$$(x_1, y_1), (x_2, y_2), \dots, (x_N, y_N)$$

where $x_i \in X$ is the i th input vector and $y_i \in Y$ is the corresponding output, find the model f that best approximates the true underlying function mapping x to y .

In that way, nonlinear regression by polynomial models (using the matlab function `polyfit`) is used to build a nonlinear function approximation that preserve the general structure of the data . The regression problem stated as a function approximation problem is the following.

From N samples (x_i, y_i) , the polynomial function

that can better predict the output value y on the basis of the regression vector x is given by :

$$y = f(x) = \text{polyfit}(x, y, n). \quad (1)$$

n is the polynomial order or degree.

The function f is defined by:

$$f(x) = a_0 + \sum_{k=1}^n w_k x^k \quad (2)$$

Depending on the degree n , these models can be used to approximate a large class of nonlinear functions. However, the number of parameters w increases with the size of X and the degree n . That clearly reflects that polynomial models suffer from the curse of dimensionality, problem known as Runge phenomenon[4]. In general, the estimation of n can be performed by stepwise regression.

To overcome this problem, an heuristic formula (3), is proposed to compute the adequate polynomial order to build an intermediate mathematical function that describes the overall structure of the data and preserve features of the distribution such as relative maxima, minima and width.

$$n = \sqrt{N \cdot (1 + r^2)} \quad (3)$$

with r = correlation coefficient (X, Y) .

To illustrate this approach, (see figure1), let us consider the following process (4) representing a noisy SISO black box non linear system :

$$y = \text{sinc}(x) + \epsilon \quad (4)$$

ϵ a vector of disturbance.

In this example ,the value of the polynomial degree computed by (3) is $n = 12$, the model is more powerful compared to the others ($n = 5, n = 7$), the overall aspect of the system is guaranteed and the error of modeling is minimized.

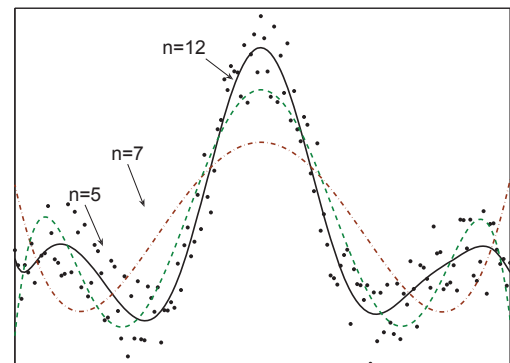


Figure 3: Nonlinear regression by polynomial models



3.2 Extracting clusters from function approximation

Analyzing the behavior of the polynomial function graph (figure 3), we can see that the data structure can be partitioned in seven substructures or clusters. This number is equal to the number of turning points, maximum and minimum, increased by unit. Consequently, we can propose the following result : finding the correct number of cluster involves the determination of the number of turning points of the approximation function described by the polynomial model.

According to the two fundamentals theorems of algebra:

Theorem 1 Any polynomial $P(x) = a_0 + \sum_{k=1}^n w_k x^k$ of degree n , with real coefficients has precisely n zeros in C .

Theorem 2 A polynomial function of degree n , can have at most $(n - 1)$ turning points.

The total number of turning points are the roots of the polynomial function $P(x)$, determined by using the command matlab *polyder*. In our case, the derived polynomial $P'(x)$ of degree $(n - 1)$ has $(n - 1)$ real or complex roots.

Assuming that α is the number of real roots and β the number of complex roots, referring to the first and second theorem, the polynomial degree and roots are bound by the following equation (5):

$$\alpha + \beta = n - 1 \quad (5)$$

Since the system is defined in the data space $(X * Y)$, we are interested thereafter only in the real roots which are included in the data input interval X . Suppose that α^* is the number of total real roots λ_i which define the turning points abscises, such as :

$$\lambda = [\lambda_1, \dots, \lambda_{\alpha^*}] \in X$$

The data input space can be partitioned in (α^*) intervals:

$$X = \underbrace{[x_1, \dots, \lambda_1]}_{X_1} \cup \underbrace{[\lambda_1, \dots, \lambda_2]}_{X_2} \cup \dots \cup \underbrace{[\lambda_{\alpha^*}, \dots, x_N]}_{X_{c^*}}$$

As a result, according to the remark made at the beginning of the paragraph, the number c^* , defined in (6), is the global number of cluster that should be used to partition the data space (7).

$$c^* = \alpha^* + 1 \quad (6)$$

$$X * Y = \bigcup_{i=1}^{c^*} \overbrace{[X_i, Y_i]}^{\text{cluster}}. \quad (7)$$

X_i and Y_i are respectively the i th sub input data interval and its corresponding sub output interval.

By applying the least square method for each group of data in a cluster, we can identify for each cluster a local linear model: $y = a_i x + b_i, i = 1, \dots, c^*$ that approximates each sub system as shown in figure 4.

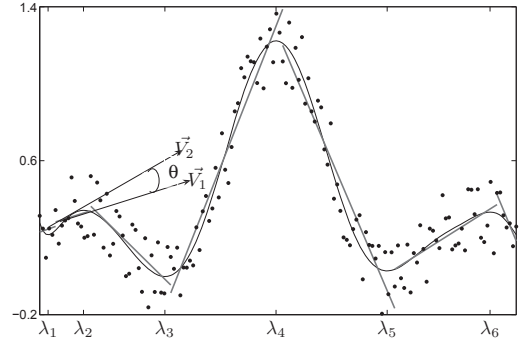


Figure 4: Partition of input data set in 7 subsets based on function approximation turning points.

3.3 Determining the number optimal of cluster

In figure 4, the two first regressed linear models present a strongly correlation. The corresponding groups of data may be merged in one cluster. In that way, for optimizing the number of clusters, we propose a clustering merging criterion based on the fusion of two consecutive line segment i and $i + 1$, given by (8) into a single cluster if their associated vectors are nearly correlated or linear. We assume that each local

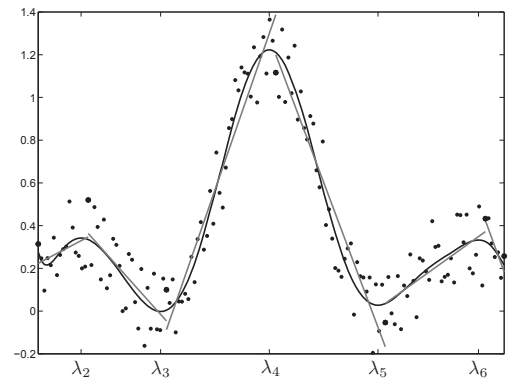


Figure 5: Reduction of the number of local models (clusters) to 6 by the merging technique.

regression model is interpreted as a vector $\vec{V}_i$ defined for $x \in X_i, y \in Y_i$.

$$y = a_i x + b_i, i = 1, \dots, c^*. \quad (8)$$

It is known, trigonometry and geometrical approach, see figure 4, that the co-linearity or correlation of two vectors is measurable by the angle :

$$\theta = \text{angle}(\vec{V}_i, \vec{V}_{i+1}), i = 1, \dots, (c^* - 1)$$

$\vec{V}_i$ is the vector corresponding to the i th local linear model.

In addition there is a relationship between this angle and the correlation of the vectors assumed representing the regressed data in each cluster $\cos \theta = r$.



The angle for which the merging criteria is validated, is taken equal to $\theta_r = 30$, this is equivalent for a correlation coefficient of data equal to $r = 0.82$, considered strongly correlation. The cluster merging algorithm is presented below:

1. Initial parameters

- $k = 0$, *Fusion number counter*.
- $\theta_r = 30$, *Validation merge criterion*.

for $i = 1$ to $(c^* - 1)$

2. Construction of vectors and angle θ .

- $\vec{V}_i \leftarrow y = a_i x + b_i, x \in X_i$.
- $\vec{V}_{i+1} \leftarrow y = a_{i+1} x + b_{i+1}, x \in X_{i+1}$.
- $\theta = \text{angle}(\vec{V}_i, \vec{V}_{i+1})$.

3. Merge criterion control.

- if $\theta \leq \theta_r$.
- *Merging intervals* $X'_i = X_i \cup X_{i+1}$.
- *Resulting local linear model* $y' = a'_i x + b'_i$.
- $k = k + 1$.

The optimal number of clusters corresponding to the optimal partition is then given by:

$$C = c^* - k. \quad (9)$$

The number of clusters determined in subsection 3.2, for the system (4), is equal to seven clusters; this number is reduced to six by applying the merging method. Figure 5 shows the new partitioning of the X data input, the sub data inputs X_1, X_2 are merged and a new sub local linear model is built to approximate the data in the resulting cluster corresponding to sub input interval $[x_1, \dots, \lambda_2]$:

$$X = \underbrace{[x_1, \dots, \lambda_2]}_{X'_1} \cup \underbrace{[\lambda_2, \dots, \lambda_3]}_{X'_2} \cup \dots \cup \underbrace{[\lambda_6, \dots, x_N]}_{X'_6}$$

X'_1 is the resulting sub input interval from the fusion of X_1 and X_2 .

Comparing the fuzzy model (figure 1) and the polynomial function (figure 3), the two models represent identically the general trend of data, this shows well, the effectiveness of the polynomial function to be used as a basis for partitioning of the data of an unknown nonlinear system.

4 Simulations results

In order to verify the results of the proposed method, we have prepared a set of noisy data generated by the function (10) taken from [9, 8] approximated from 201 points placed in equal intervals in $[-1, 1]$.

$$y = 0.6 \sin(\pi x) + 0.3 \sin(3\pi x) + 0.1 \sin(5\pi x) + \epsilon. \quad (10)$$

ϵ is a distribution added to simulate a black box nonlinear system.

The Gustafson-Kessel (GK) algorithm is applied to cluster these data set at each cluster number c from $c = 4$ to $c = 11$.

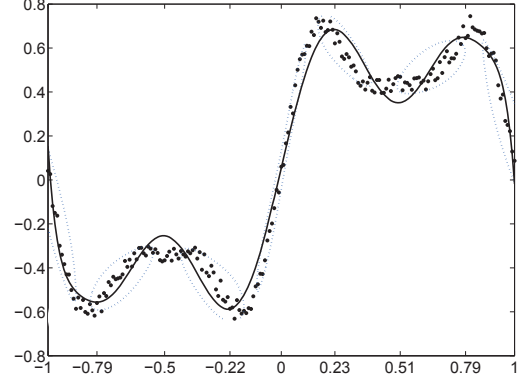


Figure 6: Result of partition of the system (10) data set : polynomial model ($n=11$), Optimal number of clusters ($c=7$).

Two validation approaches will be used to evaluate the results. The quality of clustering is tested using the fuzzy clustering toolbox to compute the so called clustering validity criteria: Partition coefficient (PC) and Classification entropy (CE). Both the two indices measure the amount of overlapping between clusters. The optimal partition can be determined by the point of the extrema of the validation indexes in dependence of the number of clusters. The optimal number of clusters is assumed equal to the local maximum of the (PC) indices and the local minimum for the (CE) [14].

The performance of the fuzzy model is measured by the VAF, which computes the percentile Variance accounted for between two signals as follows:

$$VAF = 100 \cdot \left[1 - \frac{\text{var}(y_1 - y_2)}{\text{var}(y_1)} \right]$$

y_1 is the output of the process and y_2 is the output of the fuzzy model with the same number of clusters obtained by the MATLAB fuzzy toolbox (ANFIS) and the fuzzy model identification (FMID) toolbox based on GK clustering [1]. The VAF of two equal signals is 100. If the signals differ, VAF is lower.

Figure 6 illustrates the result of our proposed method for partitioning the data based on the identified polynomial function. The polynomial degree computed by (3) is $n = 11$, number of turning points detected is 6 and the optimal number of clusters is given by (9) $c = 7$.

The performance of each clustering validity measure is given in figure 7. As seen from the figure, the (PC) and (CE) validity measures find that the optimal cluster number c is at $c = 7$.



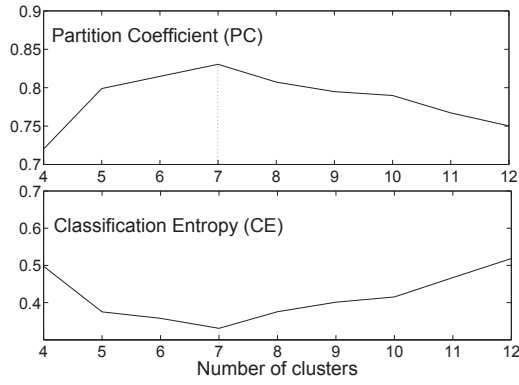


Figure 7: Performance of (PC) and (CE) clustering validation index for different number of clusters.

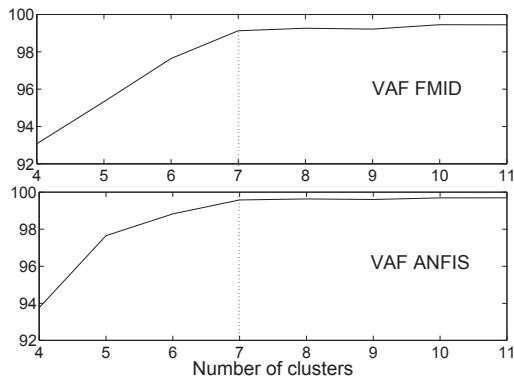


Figure 8: VAF results for different number of clusters.

In figure 8, The VAF predict $c = 7$ as the correct optimal number of clusters since there is no wide variation between the value of the VAF for the number of cluster ranging between 7 and 11. This shows applicability of our method to compute the optimal number of clusters for approximation a nonlinear black box system with a fuzzy model as shown in figure 9.

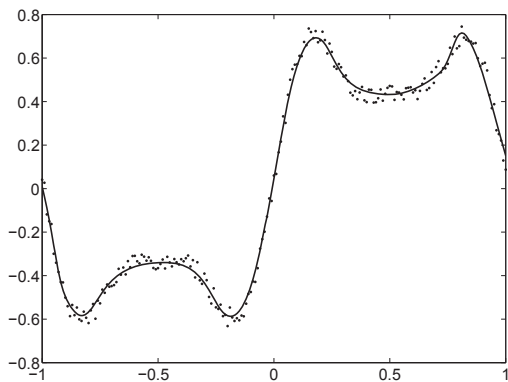


Figure 9: Comparison of the fuzzy model and the original data.

In order to verify the approach for a continuous

nonlinear system, the system (10) is taken without perturbation, our method has computed ($c = 9$) optimal clusters, the algorithm using a complementary interpolation model applied in [9] has validate the same number of interpretable TS models that capture the essence of the process under consideration.

The simulation results are illustrated in table 1.

cluster number	VAF FMID	VAF ANFIS	PC	CE
6	98.208	99.200	0.840	0.309
7	99.613	99.912	0.868	0.264
8	99.705	99.941	0.881	0.234
9	99.821	99.967	0.894	0.207
10	99.760	99.961	0.892	0.212
11	99.974	99.972	0.894	0.207

Table 1: Validation indices for the system (10) without perturbation .

5 Conclusion

The major contribution of this paper is an improved method to compute automatically the optimal number of clusters, needed as input parameter for clustering algorithms, for correctly approximating a nonlinear black box system with TS models. The simulation show that the proposed approach simplifies and makes more effective to extract optimal and legitimate TS local models from data . In addition, the data model identification used in this paper has proved to be suitable for transforming an unknown black box system into a grey box one . The approach can be extended for more complex systems, since it has proved important result for SISO nonlinear systems.

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Stability and Boundedness of Solutions of Certain Third-Order Non-Linear Delay Differential Equation

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Abstract

This article presents sufficient conditions for the uniform stability of the zero solution when $p = 0$, the uniform boundedness and uniform ultimate boundedness of all solutions in case $p \neq 0$ of the third-order nonlinear delay differential equation (1). By constructing Lyapunov functionals we achieved new contribution which includes and improves some related results existing in the relevant literature.

Keywords: *Lyapunov functional, Stability, Boundedness, Third-order delay differential equations.*

1 Introduction

Delay differential equations was encountered in the late eighteenth century by the Bernoulli's, Laplace and Condorcet. However, little was accomplished during the nineteenth century and early part of the twentieth century. During the last forty years and especially the last twenty, they have applications in domains as diverse as engineering, biology and medicine, where information transmission and/or response in control systems is not instantaneous.

Stability is a very important problem in the theory and applications of differential equations. The stability analysis of delay differential equations has received considerable attention over the decades. The stability of delay differential equations was considered in [7, 16]. Also boundedness of solutions was discussed in [16]. Later many books and papers dealt with the delay differential equations and obtained many good results, for example, [1, 3, 4, 5, 6, 8, 10], etc.

Lyapunov function is an interesting and fruitful

technique to determine the stability behavior of solutions of linear and nonlinear differential equations. This technique has gained increasing significance and has given impetus for modern development of stability theory of differential equations.

Besides, it is worth mentioning that, according our observation there are only a few papers on the same topic for certain third-order ordinary nonlinear differential equations with delay. Perhaps, the possible difficulties raising this case are due to construction of Lyapunov functional for the delay differential equations.

In fact, there is no general method to construct the Lyapunov functions and clearly, it is more difficult to construct Lyapunov functionals for higher-order differential equations with delay.

To the best of our knowledge, for a kind of third-order nonlinear delay differential equations, the stability and boundedness results have been investigated only by a few researchers, for example, In 2003, Sadek [11] obtained sufficient conditions to ensure the stability and the boundedness of solutions of the equation

$$x^{(3)} + a\ddot{x} + g(\dot{x}(t-r(t))) + f(x(t-r(t))) = p(t),$$

where a is positive constant; g, f, p are continuous functions; $g(0) = f(0) = 0$.

In 2004, Sadek [12] investigated the asymptotic stability of the zero solution of the fourth-order nonlinear delay differential equations

$$x^{(4)} + \alpha_1 \ddot{x} + \alpha_2 \ddot{x} + \alpha_3 \dot{x} + f(x(t-r)) = 0,$$

$$x^{(4)} + \alpha_1 \ddot{x} + \alpha_2 \ddot{x} + \phi(\dot{x}(t-r)) + f(x) = 0,$$

where $r, \alpha_1, \alpha_2, \alpha_3$ are positive constants; $\phi(\dot{x})$ and $f(x)$ are continuous functions and $\phi(0) = f(0) = 0$.

In 2005, Sadek [13] gave the sufficient conditions for the asymptotic stability of zero solution of third-



order delay differential equation

$$x^{(3)} + a(t)\ddot{x} + b(t)\dot{x} + c(t)f(x(t-r)) = 0,$$

where $a(t)$, $b(t)$ and $c(t)$ are positive and continuously differentiable functions on $[0, \infty)$; r is positive constant; $f(x)$ is continuous function and $f(0) = 0$.

In 2006, Tunç [14] obtained the asymptotic stability of solutions of third-order delay differential equation of the form

$$x^{(3)} + \phi(x, \dot{x})\ddot{x} + g(\dot{x}(t-r(t))) + f(x(t-r(t))) = 0,$$

and the boundedness of the equation

$$x^{(3)} + a\ddot{x} + g(\dot{x}(t-r(t))) + f(x(t-r(t))) = p(t, x, \dot{x}, \ddot{x}),$$

where $\phi(x, \dot{x})$, $g(\dot{x})$, $f(x)$ and $p(t, x, \dot{x}, \ddot{x})$ are continuous functions; $g(0) = f(0) = 0$.

In 2007, Tunç [15] investigated stability and boundedness of solutions of some nonlinear differential equations of third-order with delay of the type

$$x'''(t) + a_1x''(t) + f_2(\dot{x}(t-r(t))) + a_3x(t) = p(t, x(t), \dot{x}(t), x(t-r(t)), \dot{x}(t-r(t)), x''(t)),$$

where r is bounded delay, $0 \leq r(t) \leq \gamma$, $\dot{r}(t) \leq \beta$, $0 < \beta < 1$, β and γ are some positive constants; a_1 and a_3 are some positive constants; f_2 and p are continuous functions and $f_2(0) = 0$.

Here we consider the third-order nonlinear delay differential equation of the form:

$$x^{(3)} + f(x, \dot{x})\ddot{x} + g(x(t-r(t)), \dot{x}(t-r(t))) + h(x(t-r(t))) = p(t, x, \dot{x}, \ddot{x}, x(t-r(t)), \dot{x}(t-r(t))), \quad (1)$$

where $0 \leq r(t) \leq \gamma$, γ is positive constant which will be determined later; f, g, h and p are continuous functions; $h(0) = g(x, 0) = 0$.

Our object is to obtain sufficient conditions for the stability and for the boundedness of solutions of (1) in the cases $p = 0$ and $p \neq 0$ respectively.

The results of this paper generalize those obtained in Sadek [11], where $f(x, \dot{x})$ is a constant, $g(x(t-r(t)), \dot{x}(t-r(t)))$ was considered as a function of $\dot{x}(t-r(t))$ only and $p(t, x, \dot{x}, \ddot{x}, x(t-r(t)), \dot{x}(t-r(t)))$ was considered as a function of t only.

The remainder of this paper is organized as follows:

Section (2) gives the sufficient conditions for the stability of the zero solution of equation (1).

Section (3) shows the boundedness of all solutions of equation (1).

Section (4) presents example which illustrate the design of nonlinear controllers in feedback systems via Lyapunov's criterion. Proceeding from the knowledge of Lyapunov functions and their stability conditions the form of the nonlinearities is obtained and their effect on the overall nonlinear feedback system is studied.

2 Stability

By constructing Lyapunov functionals, we obtained very clear results which can be checked and applied easily. The following is the first main result.

Theorem 1. Let $h(0) = g(x, 0) = 0$ for all x . Suppose that there exist positive constants a, b, c, L, M, N and a_0 with $ab - c > 0$ such that

$$(i) \quad 0 \leq f(x, y) - a \leq a_0, \quad y \frac{\partial f(x, y)}{\partial x} \leq 0 \quad \text{for all } x, y;$$

$$(ii) \quad \frac{g(x, y)}{y} \geq b > 0, \quad y \neq 0;$$

$$(iii) \quad \sup \{h'(x)\} = c > 0, \quad h(x) \operatorname{sgn} x > 0 \quad \text{for } x \neq 0;$$

$$(iv) \quad r(t) \leq \gamma, \quad r'(t) \leq \beta, \quad 0 < \beta < 1;$$

$$(v) \quad |h'(x)| \leq L, \quad \left| \frac{\partial g(x, y)}{\partial x} \right| \leq M, \quad \left| \frac{\partial g(x, y)}{\partial y} \right| \leq N.$$

Then the zero solution of (1) with $p = 0$ is uniformly stable, provided that

$$\gamma < \min \left[\frac{(ab-c-2M)(1-\beta)}{\mu(L+M+N)(1-\beta)+(L+M)(1+\mu)}, \frac{(ab-c)(1-\beta)}{b\{(L+M)(1-\beta)+N(\mu-\beta+2)\}} \right],$$

where $\mu = (ab + c)/2b$.

Now we will give the stability criteria for the general autonomous delay differential system. We consider:

$$\dot{x} = f(x_t), \quad x_t = x(t + \theta), \quad -r \leq \theta \leq 0, \quad t \geq 0, \quad (2)$$

where $f: C_H \rightarrow \mathbb{R}^n$ is a continuous mapping, $f(0) = 0$, $C_H := \{\phi \in (C[-r, 0], \mathbb{R}^n) : \|\phi\| \leq H\}$ and for $H_1 < H$, there exists $L(H_1) > 0$, with $|f(\phi)| \leq L(H_1)$ when $\|\phi\| \leq H_1$.

Lemma 1.[1, 17] Let $V(\phi): C_H \rightarrow \mathbb{R}$ be a continuous functional satisfying a local Lipschitz condition, $V(0) = 0$ and functions $W_i(r)$, $(i = 1, 2)$ are wedges such that

$$(i) \quad W_1(|\phi(0)|) \leq V(\phi) \leq W_2(\|\phi\|) \text{ and}$$

$$(ii) \quad \dot{V}_{(2)}(\phi) \leq 0, \text{ for } \phi \in C_H.$$

Then the zero solution of (2) is **uniformly stable**.

Proof of Theorem 1.

We write equation (1) with $p = 0$ as the following equivalent system:

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= z, \\ \dot{z} &= -f(x, y)z - g(x, y) - h(x) \\ &\quad + \int_{t-r(t)}^t \frac{\partial g(x(s), y(s))}{\partial x} y(s) ds \\ &\quad + \int_{t-r(t)}^t \frac{\partial g(x(s), y(s))}{\partial y} z(s) ds \\ &\quad + \int_{t-r(t)}^t h'(x(s)) y(s) ds. \end{aligned} \quad (3)$$

Define the Lyapunov functional as

$$\begin{aligned} V_1(x_t, y_t, z_t) &= \mu \int_0^x h(\xi) d\xi + h(x)y \\ &\quad + \mu \int_0^y f(x, \eta) \eta d\eta + \int_0^y g(x, \eta) d\eta + \mu yz \\ &\quad + \frac{1}{2} z^2 + \lambda \int_{-r(t)}^0 \int_{t+s}^t y^2(\theta) d\theta ds \\ &\quad + \delta \int_{-r(t)}^0 \int_{t+s}^t z^2(\theta) d\theta ds, \end{aligned} \quad (4)$$



where μ, λ and δ are positive constants which will be determined later. Thus from (4) and (3) we find

$$\begin{aligned} \frac{d}{dt} V_1(x_t, y_t, z_t) = & h'(x)y^2 + \mu z^2 - \mu y g(x, y) \\ & - f(x, y)z^2 + \mu \int_0^y y \frac{\partial f(x, \eta)}{\partial x} \eta d\eta + \int_0^y y \frac{\partial g(x, \eta)}{\partial x} d\eta \\ & + (\mu y + z) \left\{ \int_{t-r(t)}^t h'(x(s))y(s)ds \right. \\ & \quad + \int_{t-r(t)}^t \frac{\partial g(x(s), y(s))}{\partial x} y(s)ds \\ & \quad \left. + \int_{t-r(t)}^t \frac{\partial g(x(s), y(s))}{\partial y} z(s)ds \right\} \\ & + \lambda y^2 r(t) - \lambda(1-r'(t)) \int_{t-r(t)}^t y^2(\theta)d\theta \\ & + \delta z^2 r(t) - \delta(1-r'(t)) \int_{t-r(t)}^t z^2(\theta)d\theta. \end{aligned} \quad (5)$$

From (i) and (v) we find

$$\begin{aligned} \frac{d}{dt} V_1(x_t, y_t, z_t) \leq & h'(x)y^2 + \mu z^2 - \mu y g(x, y) + M y^2 \\ & - f(x, y)z^2 + (\mu y + z) \left\{ (M + L) \int_{t-r(t)}^t y(s)ds \right. \\ & \quad \left. + N \int_{t-r(t)}^t z(s)ds \right\} \\ & + \lambda y^2 r(t) - \lambda(1-r'(t)) \int_{t-r(t)}^t y^2(\theta)d\theta \\ & + \delta z^2 r(t) - \delta(1-r'(t)) \int_{t-r(t)}^t z^2(\theta)d\theta. \end{aligned} \quad (6)$$

From (i), (ii), (iii), (iv) and by using $2uv \leq u^2 + v^2$ we obtain

$$\begin{aligned} \frac{d}{dt} V_1 \leq & - \left\{ \mu b - c - M - \frac{\mu(L+M+N)}{2} \gamma - \lambda \gamma \right\} y^2 \\ & - \left\{ a - \mu - \frac{L+M+N}{2} \gamma - \delta \gamma \right\} z^2 \\ & + \left\{ \frac{L+M}{2} + \frac{\mu(L+M)}{2} - (1-\beta)\lambda \right\} \int_{t-r(t)}^t y^2(s)ds \\ & + \left\{ \frac{N}{2} + \frac{\mu N}{2} - (1-\beta)\delta \right\} \int_{t-r(t)}^t z^2(s)ds. \end{aligned} \quad (7)$$

We can take $\mu = \frac{ab+c}{2b} > 0$, $\lambda = \frac{(L+M)(1+\mu)}{2(1-\beta)} > 0$, $\delta = \frac{N(1+\mu)}{2(1-\beta)} > 0$ so that

$$\begin{aligned} \frac{d}{dt} V_1 \leq & - \left\{ \frac{ab-c}{2} - M \right. \\ & \quad \left. - \frac{\mu(L+M+N)(1-\beta) + (L+M)(\mu+1)}{2(1-\beta)} \gamma \right\} y^2 \\ & - \left\{ \frac{ab-c}{2b} - \frac{(L+M)(1-\beta) + N(\mu-\beta+2)}{2(1-\beta)} \gamma \right\} z^2. \end{aligned}$$

Therefore if

$$\gamma < \min \left[\frac{(ab-c-2M)(1-\beta)}{\mu(L+M+N)(1-\beta) + (L+M)(1+\mu)}, \frac{(ab-c)(1-\beta)}{b\{(L+M)(1-\beta) + N(\mu-\beta+2)\}} \right],$$

we have

$$\frac{d}{dt} V_1(x_t, y_t, z_t) \leq -\alpha(y^2 + z^2), \quad \text{for some } \alpha > 0. \quad (8)$$

From (iii) by considering $h(x)sgnx > 0$ for $x \neq 0$, we obtain

$$\begin{aligned} V_1(x_t, y_t, z_t) \geq & \mu \int_0^x h(\xi)d\xi + h(x)y + \frac{1}{2}\mu a y^2 \\ & + \frac{1}{2}b y^2 + \mu y z + \frac{1}{2}z^2 + \lambda \int_{-r(t)}^0 \int_{t+s}^t y^2(\theta)d\theta ds \\ & + \delta \int_{-r(t)}^0 \int_{t+s}^t z^2(\theta)d\theta ds \\ = & \frac{1}{2b}(by + h(x))^2 + \frac{1}{2}(\mu y + z)^2 + \frac{1}{2}\mu(a - \mu)y^2 \\ & + \frac{1}{2b y^2} \left[4 \int_0^x h(\xi) \left\{ \int_0^y (\mu b - h'(\xi))\eta d\eta \right\} d\xi \right] \\ & + \lambda \int_{-r(t)}^0 \int_{t+s}^t y^2(\theta)d\theta ds \\ & + \delta \int_{-r(t)}^0 \int_{t+s}^t z^2(\theta)d\theta ds, \quad \text{for } y \neq 0. \end{aligned}$$

But $a - \mu = (ab - c)/2b > 0$, $\mu b - h'(x) \geq (ab - c)/2 > 0$ and since the integrals $\lambda \int_{-r(t)}^0 \int_{t+s}^t y^2(\theta)d\theta ds$ and $\delta \int_{-r(t)}^0 \int_{t+s}^t z^2(\theta)d\theta ds$ are non negative, then we obtain

$$V_1(x_t, y_t, z_t) \geq \frac{1}{2b}(by + h(x))^2 + \frac{1}{2}(\mu y + z)^2 + \frac{1}{2}\mu(a - \mu)y^2, \quad \text{for } y \neq 0. \quad (9)$$

From (i), (iv), (v) and by using the mean-value theorem we find

$$\begin{aligned} V_1(x_t, y_t, z_t) \leq & \mu \int_0^x L\xi d\xi + L|xy| + \mu \int_0^y (a_0 + a)\eta d\eta \\ & + \int_0^y N\eta d\eta + \frac{1}{2}z^2 + \mu|yz| \\ & + \lambda \int_{t-r(t)}^t (\theta - t + r(t))y^2(\theta)d\theta \\ & + \delta \int_{t-r(t)}^t (\theta - t + r(t))z^2(\theta)d\theta \\ \leq & \frac{(\mu+1)L}{2}\|x\|^2 + \frac{L+N+\mu(a+a_0+1)+\lambda\gamma^2}{2}\|y\|^2 \\ & + \frac{\mu+1+\delta\gamma^2}{2}\|z\|^2. \end{aligned} \quad (10)$$

Then from (8), (9) and (10) $V_1(x_t, y_t, z_t)$ satisfies the conditions of Lemma 1.

Thus the proof of Theorem 1 is now complete.

3 Boundedness

The following is the second main result.

Theorem 2. Suppose, further to the conditions of Theorem 1, that there exist constants $c_0 > 0$, $m > 0$ and $m_0 > 0$ such that $\frac{h(x)}{x} \geq c_0$ for $x \neq 0$, $p \leq m + m_0(|x| + |y| + |z|)$. Then every solution of equation (1) is uniformly bounded and uniformly ultimately bounded, provided that,

$$\gamma < \min \left\{ \frac{2c_0}{L+M+N}, \frac{(ab-c-2M)(1-\beta)}{(L+M)(1+\mu+ab-c+a+a^2) + (L+M+N)(\mu+a^2)(1-\beta)}, \frac{(ab-c)(1-\beta)}{bN(1+\mu+ab-c+a+a^2) + b(L+M+N)(1+a)(1-\beta)} \right\},$$

where $\mu = (ab + c)/2b$.

Now we will consider a system of delay differential equations

$$\dot{\bar{x}} = F(t, \bar{x}_t), \quad \bar{x}_t = \bar{x}(t + \theta), \quad -r \leq \theta \leq 0, \quad (11)$$

where $F: \mathbb{R} \times C \rightarrow \mathbb{R}^n$ is a continuous mapping, and takes bounded sets into bounded sets.



The following Lemma is a well-known result obtained by Burton [2].

Lemma 1. Let $V(t, \phi) : \mathbb{R} \times C \rightarrow \mathbb{R}$ be continuous and locally Lipschitz in ϕ . If

$$(i) \quad W(|\bar{x}(t)|) \leq V(t, \bar{x}_t) \leq W_1(|\bar{x}(t)|) + W_2\left(\int_{t-r}^t W_3(|\bar{x}(s)|)ds\right), \text{ and}$$

(ii) $\dot{V}_{(11)}(t, x_t) \leq -W_3(|\bar{x}(t)|) + M$, for some $M > 0$ where $W(r)$, $W_i(r)$, ($i = 1, 2, 3$) are wedges. Then the solutions of (11) are uniformly bounded and uniformly ultimately bounded for bound B .

Proof of Theorem 2.

Now we will consider the boundedness of the solutions of equation (1) by using this lemma.

We assume that $p(t, x, \dot{x}, \ddot{x}, x(t-r(t)), \dot{x}(t-r(t)))$ is bounded with bound $m + m_0(|x| + |y| + |z|)$ and the conditions of Theorem 1 hold. Equation (1) has an equivalent system

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= z, \\ \dot{z} &= -f(x, y)z - g(x, y) - h(x) \\ &\quad + \int_{t-r(t)}^t \frac{\partial g(x(s), y(s))}{\partial x} y(s)ds \\ &\quad + \int_{t-r(t)}^t \frac{\partial g(x(s), y(s))}{\partial y} z(s)ds \\ &\quad + \int_{t-r(t)}^t h'(x(s))y(s)ds \\ &\quad + p(t, x, y, z, x(t-r(t)), y(t-r(t))). \end{aligned} \quad (12)$$

Consider the Lyapunov functional

$$V = V_1(x_t, y_t, z_t) + V_2(x_t, y_t, z_t), \quad (13)$$

where $V_1(x_t, y_t, z_t)$ is defined as (4) and $V_2(x_t, y_t, z_t)$ is defined as

$$\begin{aligned} V_2(x_t, y_t, z_t) &= a^2 \int_0^x h(\xi)d\xi + a^2 \int_0^y f(x, \eta)\eta d\eta \\ &\quad + \frac{b}{2}(ab-c)x^2 + ah(x)y + \frac{c}{2}y^2 \\ &\quad + (ab-c)x(z+ay) + \frac{a}{2}z^2 + a^2zy. \end{aligned} \quad (14)$$

From (4), (7) and (12) we find

$$\begin{aligned} \frac{d}{dt} V_1 &\leq -\left\{ \frac{ab-c}{2} - M - \frac{\mu(L+M+N)}{2}\gamma - \lambda\gamma \right\} y^2 \\ &\quad - \left\{ \frac{ab-c}{2b} - \frac{L+M+N}{2}\gamma - \delta\gamma \right\} z^2 \\ &\quad + \left\{ \frac{L+M}{2} + \frac{\mu(L+M)}{2} - (1-\beta)\lambda \right\} \int_{t-r(t)}^t y^2(s)ds \\ &\quad + \left\{ \frac{N}{2} + \frac{\mu N}{2} - (1-\beta)\delta \right\} \int_{t-r(t)}^t z^2(s)ds \\ &\quad + (\mu|y| + |z|)\{m + m_0(|x| + |y| + |z|)\}. \end{aligned} \quad (15)$$

Also from (14), (12) and by using the conditions in Theorem 1, we obtain

$$\begin{aligned} \frac{d}{dt} V_2(x_t, y_t, z_t) &\leq -(ab-c)xh(x) \\ &\quad + (ab-c)|x|\{m + m_0(|x| + |y| + |z|)\} \\ &\quad + a|z + ay|\{m + m_0(|x| + |y| + |z|)\} \\ &\quad + (ab-c)xN \int_{t-r(t)}^t z(s)ds \\ &\quad + (ab-c)x(M+L) \int_{t-r(t)}^t y(s)ds \\ &\quad + a(z+ay)N \int_{t-r(t)}^t z(s)ds \\ &\quad + a(z+ay)(M+L) \int_{t-r(t)}^t y(s)ds. \end{aligned} \quad (16)$$

Since $r(t) \leq \gamma$, and by using $2uv \leq u^2 + v^2$ we have

$$\begin{aligned} \frac{d}{dt} V_2(x_t, y_t, z_t) &\leq -(ab-c)xh(x) \\ &\quad + (ab-c)|x|\{m + m_0(|x| + |y| + |z|)\} \\ &\quad + a|z + ay|\{m + m_0(|x| + |y| + |z|)\} \\ &\quad + \frac{ab-c}{2}(L+M+N)\gamma x^2 + \frac{a^2}{2}(L+M+N)\gamma y^2 \\ &\quad + \frac{a}{2}(L+M+N)\gamma z^2 + \left\{ \frac{ab-c}{2}(L+M) \right. \\ &\quad \left. + \frac{a}{2}(L+M) + \frac{a^2}{2}(L+M) \right\} \int_{t-r(t)}^t y^2(s)ds \\ &\quad + \left(\frac{ab-c}{2}N + \frac{a}{2}N + \frac{a^2}{2}N \right) \int_{t-r(t)}^t z^2(s)ds. \end{aligned} \quad (17)$$

Therefore from (13), (15) and (17), we obtain

$$\begin{aligned} \frac{d}{dt} V &\leq -\left\{ \frac{ab-c}{2} - M - \lambda\gamma - \frac{(\mu+a^2)(L+M+N)}{2}\gamma \right\} y^2 \\ &\quad - \left\{ \frac{ab-c}{2b} - \frac{L+M+N}{2}\gamma - \delta\gamma - \frac{a(L+M+N)}{2}\gamma \right\} z^2 \\ &\quad - (ab-c)xh(x) + \frac{ab-c}{2}(L+M+N)\gamma x^2 \\ &\quad + \left\{ \frac{ab-c}{2}(L+M) + \frac{a(a+1)}{2}(L+M) \right. \\ &\quad \left. + \frac{(\mu+1)(L+M)}{2} - (1-\beta)\lambda \right\} \int_{t-r(t)}^t y^2(s)ds \\ &\quad + \left\{ \frac{ab-c}{2}N + \frac{aN}{2} + \frac{a^2N}{2} + \frac{\mu N}{2} + \frac{N}{2} \right. \\ &\quad \left. - (1-\beta)\delta \right\} \int_{t-r(t)}^t z^2(s)ds \\ &\quad + (ab-c)|x|(m + m_0) \\ &\quad + (a|z + ay| + \mu|y| + |z|)(m + m_0). \end{aligned} \quad (18)$$

Suppose that $\frac{h(x)}{x} \geq c_0$ for $x \neq 0$. Then

$$\begin{aligned} \frac{d}{dt} V &\leq -\left\{ (ab-c)(c_0 - \frac{L+M+N}{2}\gamma) \right\} x^2 \\ &\quad - \left[\frac{ab-c}{2} - M - \left\{ \lambda + \frac{1}{2}(L+M+N)(\mu+a^2) \right\} \gamma \right] y^2 \\ &\quad - \left[\frac{ab-c}{2b} - \left\{ \delta + \frac{1}{2}(L+M+N)(1+a) \right\} \gamma \right] z^2 \\ &\quad - \left\{ (1-\beta)\lambda - \frac{L+M}{2}(1+\mu+ab-c+a+a^2) \right\} \\ &\quad \int_{t-r(t)}^t y^2(s)ds - \left\{ (1-\beta)\delta - \frac{N}{2}(1+\mu+ab-c \right. \\ &\quad \left. + a+a^2) \right\} \int_{t-r(t)}^t z^2(s)ds + (ab-c)m|x| \\ &\quad + (a^2m + \mu m)|y| + (am + m)|z| \\ &\quad + (ab-c)|x|m_0(|x| + |y| + |z|) \\ &\quad + (a^2 + \mu)|y|m_0(|x| + |y| + |z|) \\ &\quad + (a+1)|z|m_0(|x| + |y| + |z|). \end{aligned}$$



Let $\lambda = \frac{L+M}{2(1-\beta)} (1 + \mu + ab - c + a + a^2) > 0$,
 $\delta = \frac{N}{2(1-\beta)} (1 + \mu + ab - c + a + a^2) > 0$. When

$$\gamma < \min \left\{ \frac{2c_0}{L+M+N}, \frac{(ab-c-2M)(1-\beta)}{(L+M)(1+\mu+ab-c+a+a^2)+(L+M+N)(\mu+a^2)(1-\beta)}, \frac{(ab-c)(1-\beta)}{bN(1+\mu+ab-c+a+a^2)+b(L+M+N)(1+a)(1-\beta)} \right\},$$

where $\mu = (ab + c)/2b$.

Then we can take

$$k = m_0 \max (ab - c, a^2 + \mu, a + 1)$$

and

$$k (|x| + |y| + |z|)^2 \leq 3k (x^2 + y^2 + z^2)$$

We get

$$\begin{aligned} \frac{d}{dt}V &\leq -\alpha(x^2 + y^2 + z^2) + k\alpha(|x| + |y| + |z|) \\ &= -\frac{\alpha}{2}(x^2 + y^2 + z^2) \\ &\quad -\frac{\alpha}{2}\{(|x| - k)^2 + (|y| - k)^2 + (|z| - k)^2\} + \frac{3\alpha}{2}k^2 \\ &\leq -\frac{\alpha}{2}(x^2 + y^2 + z^2) + \frac{3\alpha}{2}k^2, \text{ for some } \alpha, k > 0. \end{aligned}$$

Then condition (ii) of Lemma 2 is satisfied by taking $W_3(r) = \frac{\alpha}{2}r^2$ and $M = \frac{3\alpha}{2}k^2$.

From (i) and (iii) we have

$$\begin{aligned} V_2(x_t, y_t, z_t) &\geq a^2 \int_0^x h(\xi)d\xi + ah(x)y + \frac{1}{2}a^3y^2 \\ &\quad + \frac{b}{2}(ab - c)x^2 + \frac{c}{2}y^2 + (ab - c)x(z + ay) \\ &\quad + \frac{a}{2}z^2 + a^2zy \\ &= \frac{a^2}{2cy^2} \left[4 \int_0^x h(\xi) \left\{ \int_0^y (c - h'(\xi))\eta d\eta \right\} d\xi \right] \\ &\quad + \frac{1}{2c}(cy + ah(x))^2 + \frac{c}{2b}(z + ay)^2 \\ &\quad + \frac{ab-c}{2b}\{bx + (z + ay)\}^2. \end{aligned} \quad (19)$$

By using the similar techniques of (10) we obtain

$$\begin{aligned} V(x_t, y_t, z_t) &\leq \left\{ \frac{(\mu+a+a^2+1)L}{2} + \frac{(ab-c)(b+1)}{2} \right\} x^2 \\ &\quad + \left\{ \frac{(a+a_0)(\mu+a^2)}{2} + \frac{(a+1)L+\mu+N+a^2}{2} + \frac{(a+a^2)(ab-c)}{2} \right\} y^2 \\ &\quad + \left\{ \frac{\mu+a+a^2+1}{2} + \frac{(a+1)(ab-c)}{2} \right\} z^2 \\ &\quad + \frac{\gamma\beta}{\alpha} \left\{ \int_{t-r(t)}^t \frac{\alpha}{2} (\|x\|^2 + \|y\|^2 + \|z\|^2) ds \right\}. \end{aligned} \quad (20)$$

where $\beta = \max(\lambda, \delta)$.

Therefore from (9), (19) and (20) the functional $V(x_t, y_t, z_t)$ satisfies condition (i) of Lemma 2. Thus the proof of Theorem 2 is now complete.

4 Case of Study

Lyapunov methods have been used by control system designers to obtain stabilizing feedback controllers for nonlinear system.

We apply Lyapunov's direct method to the stability study of nonlinear feedback control systems.

A direct feedback control system is shown in figure 1. (see Ku, Mekel and Su [9]). The system to be controlled is denoted by its transfer function $G_1(s)$. The controller is denoted by N in parallel with an amplifier which gives gain K_1 . The controller may contain linear as well as nonlinear functions of the variable x and its derivatives. In an autonomous system, the reference input is zero ($r = 0$). Hence the output C is equal to $-x$, where x denotes error or actuating signal. The input to $G_1(s)$ is denoted by m, the manipulated signal. As shown m is the sum of K_1x and the output of the controller N, which may be denoted by $n(t)$.

Let

$$G_1(s) = \frac{1}{s(s^2 + \alpha_1 s + \alpha_2)}. \quad (21)$$

The amplifier has a gain K_1 . The differential equation of the closed-loop linear system is given by

$$x^{(3)} + \alpha_1 \ddot{x} + \alpha_2 \dot{x} + K_1 x(t - r(t)) = 0, \quad (22)$$

controllers with delay.

Let the nonlinear controller give its output as

$$n(t) = f_1(x, \dot{x}) \ddot{x} + g_1(\dot{x}) + h_1(x(t - r(t))). \quad (23)$$

Then

$$m(t) = K_1 x(t - r(t)) + f_1(x, \dot{x}) \ddot{x} + g_1(\dot{x}) + h_1(x(t - r(t))). \quad (24)$$

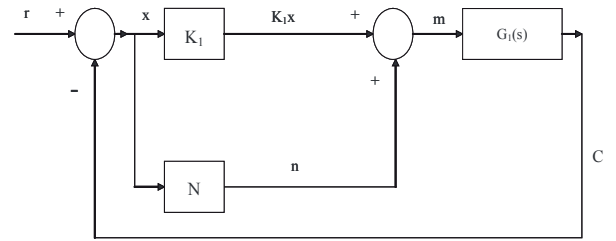


Figure 1: Block diagram of nonlinear system

The differential equation for the overall nonlinear control system with direct feedback is

$$x^{(3)} + (\alpha_1 + f_1(x, \dot{x})) \ddot{x} + \alpha_2 \dot{x} + g_1(\dot{x}) + h_1(x(t - r(t))) + K_1 x(t - r(t)) = 0. \quad (25)$$

If we let

$f(x, \dot{x}) = \alpha_1 + f_1(x, \dot{x})$, $g(x, \dot{x}) = \alpha_2 \dot{x} + g_1(\dot{x})$,
 $h(x(t - r(t))) = K_1 x(t - r(t)) + h_1(x(t - r(t)))$,
 we obtain the following nonlinear delay differential equation of the third-order

$$x^{(3)} + f(x, \dot{x}) \ddot{x} + g(x, \dot{x}) + h(x(t - r(t))) = 0. \quad (26)$$

We note that equation (26) is a special case of equation (1) and the functions f, g, h satisfies the conditions of Theorem 1, provided that $g_1(0) = 0$ and $\frac{g_1(y)}{y} \geq b$, $y \neq 0$.



5 Conclusion

The most effective method to determine the stability behavior and boundedness of solutions of linear and non-linear differential equations is the Lyapunov's direct method. The major advantage of this method is that stability and boundedness in the large can be obtained without any prior knowledge of solutions. Today, this method is widely recognized as an excellent tool not only in the study of differential equations but also in the theory of control systems, dynamical systems, systems with time lag, power system analysis, time varying non-linear feedback systems, and so on. This paper gives an introductory study on the stability, boundedness and design of nonlinear control systems of third-order delay differential equation. It shows that Lyapunov's criterion can be applied with advantage from the control engineer's point of view to nonlinear feedback systems.

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Fuzzy Logic PI Controller for a class of MIMO Nonlinear Systems

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Abstract

This paper addresses the problem of designing a fuzzy logic PI controller for a class of Multi-input Multi-output (MIMO) systems having a polynomial input nonlinearity. The MIMO system is decomposed into Multi-input Single-output (MISO) subsystems. By this decomposition, the identification, decoupling and control become more performing and easy. We propose to approximate all the interactions between the loops, the nonlinearities and disturbances using the polynomial approach. The control is conducted at the level of each subsystem and the decoupling is easily achieved by compensation of the cross-coupling. Then the control of a MIMO system is transformed into the control of a set of coupled nonlinear MISO subsystems. A good performance for tracking is obtained using fuzzy logic PI controller. The validity and robustness of the proposed approach is tested on a simulation example.

Keyword: MIMO systems, Adaptive control, Nonlinear systems, Decoupling, Fuzzy logic controller

1. Introduction

The majority of process industries are nonlinear and multivariable. Due the interconnections, dead time and nonlinearity, the control of these systems become difficult. It is obvious that the difficulty of MIMO systems control is how to overcome the coupling effects among each degree of freedom. To obtain good performance, coupling effect cannot be neglected. Hence SISO system control scheme is not easy to implement on complicated MIMO systems. Therefore, intelligent control strategy is gradually drawing attention. A multivariable decoupling control strategy should be adapted to obtain good performance for tracking purpose. A multivariable adaptive control ensuring decoupling has been the topic of several recent papers. One approach is a parameterization method to avoid the interactor matrix [1,2]. Another approach is to use a precompensator in cascade with the plant so that we obtain a diagonal interactor. Thus, the existence of such precompensator is characterized by a priori knowledge on the relative degree of each element of the plant, while the parameter values are unknown. However this parameterization is

quite complicated. Different decoupling approaches can be found in the literatures [1, 3, 4, 5, 6, 22].

However, all these approaches are computationally intensive and can not be applied to MIMO nonlinear systems subject to unmodelled dynamics.

Fuzzy logic control methodologies have emerged in recent years as a promising. Fuzzy logic control has found extensive applications for plants that are complex and ill-defined. Based on the universal approximation theorem [17] several stable adaptive fuzzy control schemes have developed to incorporate the expert knowledge systematically for a class of nonlinear systems with unknown dynamics and disturbances [23, 24, 25]. The MIMO systems usually process characteristics of nonlinear dynamics coupling [2, 5, 11, 12] and the difficulty is how to overcome the coupling effects among each degree of freedom and assuring tracking performance. Due to the above problems, we are motivated to design a fuzzy logic controller and an auxiliary control to compensate the interconnections and nonlinearity terms.

In this paper, the multivariable control design is transformed into a monovariable one by considering the MIMO system as composed of MISO subsystems. Each MISO subsystem will be controlled independently of the others. The advantage of this approach is beneficial in the sense that it allows the direct practical implementation for a large class of multivariable control systems and avoids the burden computational of the global approach. A tuned fuzzy logic controller FLC is designed according to optimize the parameters of the fuzzy system. To predict all the interactions, the nonlinearities and disturbances, we propose to use the Chebyshev polynomials approach. This prediction is taken into account in the local control design and allows to convert the nonlinear problem into a linear one. We note that discrete orthogonal polynomials have been applied for system analysis, parameter identification and optimal control with some success [7,8,9].

The paper is organized as follows: section 2 describes the problem statement. Estimation parameter algorithm is given in section 3. The structure of the fuzzy logic PI controller is detailed in section 4. In section 5, simulation results are given to test the validity of the approach.



2. Problem formulation

We consider a MIMO discrete time system having a polynomial input nonlinearity represented by the following difference equation [10]:

$$A(q^{-1})y(k) = B_d(q^{-1})Z(k) + \xi(k) \quad (1)$$

Where

$$q^{-1} \text{ is the back shift operator : } q^{-1}(x(k)) = x(k-1).$$

$A(q^{-1})$ and $B_d(q^{-1})$ are polynomials matrix defined by:

$$A(q^{-1}) = \text{diag}[A_i(q^{-1})]$$

$$A_i(q^{-1}) = \sum_{r=0}^{n_a(i)} a_{ir} q^{-r}, \quad a_{i0} = 1 \quad (2)$$

$$B_d(q^{-1}) = [q^{-d_{ij}} B_{ij}(q^{-1})] \quad (3)$$

$$B_{ij}(q^{-1}) = \sum_{r=0}^{n_b(i,j)} b_{ijr} q^{-r}, \quad b_{ij0} \neq 0$$

d_{ii} is the time delay between the input $u_i(k)$ and the output $y_i(k)$.

d_{ij} is the time delay between the input $u_j(k)$ and the output $y_i(k)$.

$y(k) \in R^n$, $Z(k) \in R^n$ and $\xi(k) \in R^n$ are respectively the output, nonlinear input and the disturbances vector.

The non linearity is assumed to be represented as follows [11]:

$$Z_i(t) = f_{i0} + f_{i1}u_i(k) + \dots + f_{ip}u_i^{p_i-1}(k) \quad (4)$$

The MIMO system (1) can be decomposed into n MISO subsystems. The identification, decoupling and control will be conducted at the level of each subsystem. Thus, the computational load is reduced.

Equation (1) becomes:

$$A_i(q^{-1})y_i(k) = q^{-d_{ii}} B_{ii}(q^{-1})Z_i(k) + \sum_{\substack{j=1 \\ j \neq i}}^n q^{-d_{ij}} B_{ij}(q^{-1})Z_j(k) + \xi_i(k) \quad (5)$$

Where $u_i(k)$ and $y_i(k)$ are respectively the i^{th} scalar input and scalar output of the system at time instant k .

The proposed control law is composed of two terms. The first is a PI regulator synthesized using a fuzzy logic technique which allows a good tracking. The second term is a compensator, where the structure is obtained from the result of the predictor of the function $H_i(\cdot)$ defined by (9).

Thus, equation (5) becomes:

$$A_i(q^{-1})y_i(k) = q^{-d_{ii}} f_{i1} B_{ii}(q^{-1})u_i(k) + q^{-d_{ii}} B_{ii}(q^{-1})[f_{i0} + f_{i2}u_i^2(k) + \dots + f_{ip}u_i^{p_i-1}(k)] \quad (6)$$

$$+ \sum_{\substack{j=1 \\ j \neq i}}^n q^{-d_{ij}} B_{ij}(q^{-1})Z_j(k) + \xi_i(k)$$

The objective is to cause the system output $y_i(k)$ to track a bounded reference sequence $y_{im}(k)$ using the fuzzy logic PI controller. However, it is hardly ensured when the system is nonlinear multivariable with disturbances.

The system (6) can be written as:

$$A_i(q^{-1})y_i(k) = q^{-d_{ii}} B_{ii}^*(q^{-1})u_i(k) + H_i(\cdot) \quad (7)$$

Where

$$B_{ii}^*(q^{-1}) = f_{i1} B_{ii}(q^{-1}) \quad (8)$$

$H_i(\cdot)$ is a non linear function of $u_i(k)$ and $u_j(k)$.

$$H_i(\cdot) = q^{-d_{ii}} f_{i0} B_{ii}(q^{-1}) + q^{-d_{ii}} B_{ii}(q^{-1}) \sum_{k=2}^{p_i-1} f_{ik} u_i^k(k) + \sum_{\substack{j=1 \\ j \neq i}}^n q^{-d_{ij}} B_{ij}(q^{-1})Z_j(k) + \xi_i(k) \quad (9)$$

We propose that $H_i(\cdot)$ will be predicted in real time using Chebyshev orthogonal polynomials. This prediction converts the nonlinear problem into a linear one.

In general, the time function $f_i(t)$ which is square integrable in the time interval $0 \leq t \leq T_f$, can be expanded into the orthogonal polynomials. In this paper, we use the Chebyshev polynomials to predict $f_i(t)$ in real time as:

$$f_i(t) = \sum_{k=0}^{\infty} f_{ik} T_k(t) \quad (10)$$

which contains an infinite number of terms. In practical applications, only a finite number should be used. Thus, approximation gives:

$$f_i(t) = \sum_{k=0}^m f_{ik} T_k(t) + \sigma_i(t) \quad (11)$$

$$= f_i^T T(t) + \sigma_i(t)$$

where $\sigma_i(t)$ is the truncation error.

The unknown function $H_i(\cdot)$ contains the nonlinear terms, cross-coupling and disturbances. Here, we propose to estimate $H_i(\cdot)$ in real time by projecting it on a set of Chebyshev orthogonal polynomials. The estimate of $H_i(\cdot)$ will be used in the control design in order to synthesise a compensator.

Based on Chebyshev approximation of $H_i(\cdot)$, the system described by equation (7) becomes:



$$A_i(q^{-1})y_i(k) = q^{-d_{ii}}B_{ii}^*(q^{-1})u_i(k) + h_i^T T(k) + \sigma_i(k) \quad (12)$$

Where $\sigma_i(k)$ is the truncation error.

Remark: concerning the expansion order of the Chebyshev polynomials.

From equation (11), the question which naturally arises is how should the expansion order m be chosen? If the problem was in a framework of signal analysis, the expansion order m should be large enough for good signal approximation (theoretically $m \rightarrow \infty$). In the present context (from the control point of view), this order is chosen to be minimal $m = 3$. This is because equation (11) is used as a prediction model. It is known that augmenting the order m to $m + q$ gives a parameter vector (equation (13)) with q extra parameters to be estimated, which increases the convergence time.

The advantage of Chebyshev approximation is that, its coefficients are bounded by the maximum value of the function to be approximated. Besides, the magnitudes of the coefficients decrease rapidly. In General, the Chebyshev series has faster convergence rate than other series using orthogonal functions [12].

Assumptions

A.1 The time d_{ii} delay is known.

A.2 $A_i(q^{-1})$ and $B_{ii}^*(q^{-1})$ are coprime.

3. Parameter Estimation

The system described by equation (12) can also be written as:

$$y_i(k) = \theta_i^T \phi_i(k-1) + \sigma_i(k) \quad (13)$$

where:

$$\begin{aligned} \phi_i(k-1) = & [-y_i(k-1), -y_i(k-2), \dots, -y_i(k-n_a), \\ & u_i(k-d_{ii}), u_i(k-d_{ii}-1), \dots, u_i(k-d_{ii}-n_b), \\ & T_0(k), T_1(k), \dots, T_{m-1}(k)] \end{aligned}$$

and

$$\theta_i = [a_{i1}, a_{i2}, \dots, a_{in_a}, b_{ii0}^*, b_{ii1}^*, \dots, b_{iin_b}^*,$$

$$h_{i0}, h_{i1}, \dots, h_{im-1}]$$

m is the expansion order of the Chebyshev polynomials. Based on parameterization (13), the identification algorithm giving estimates $\hat{\theta}_i(k)$ of θ_i can be obtained using the normalized least-squares algorithm [13].

$$\hat{\theta}_i(k) = \hat{\theta}_i(k-1) + \frac{P_i(k-1)\bar{\phi}_i(k-1)\bar{e}_i(k)}{\lambda_i(k) + \bar{\phi}_i^T(k-1)P_i(k-1)\bar{\phi}_i(k-1)} \quad (14)$$

$$\bar{e}_i(k) = \bar{y}_i(k) - \bar{\phi}_i^T(k-1)\hat{\theta}_i(k-1) \quad (15)$$

$$P_i(k) = P_i(k-1) - \frac{P_i(k-1)\bar{\phi}_i(k)\bar{\phi}_i^T(k)P_i(k-1)}{\lambda_i(k) + \bar{\phi}_i^T(k)P_i(k-1)\bar{\phi}_i(k-1)} \quad (16)$$

with $P_i(0) = \alpha I$, $0 < \alpha < \infty$ and I is the identity matrix.

$$\lambda_i(k) = \lambda_{i0}\lambda_i(k-1) + (1 - \lambda_{i0})$$

with $\lambda_{i0} = 0.95$ and $\lambda_i(0) = 0.96$.

$$\bar{y}_i(k) = \frac{y_i(k)}{\eta_i(k-1)} \quad (17)$$

$$\bar{\phi}_i(k-1) = \frac{\phi_i(k-1)}{\eta_i(k-1)} \quad (18)$$

A.3 There exists a small positive scalar ω_i such that

$$\frac{|\sigma_i(k)|}{\eta_i(k)} \leq \omega_i \text{ with } \eta_i \text{ is a normalizing signal.}$$

The proprieties which determine the parameter adaptation algorithm in adaptive control context are described in detail in [15].

4. Structure of the controller

Together with the structure of Figure 1, the proposed control structure is given as:

$$R_i(q^{-1})v_i(k) + G_i(q^{-1})y_i(k) = A_{im}(q^{-1})y_{im}(k) \quad (19)$$

where:

$v_i(k) = u_i(k) + u_{ic}(k)$ et $A_{im}(q^{-1})$ is a stable polynomial.

$u_i(k)$ is synthesised from a PI control law using fuzzy logic technique and $u_{ic}(k)$ is the auxiliary control given in the sense to compensate the nonlinear effect, the cross-coupling and disturbances.

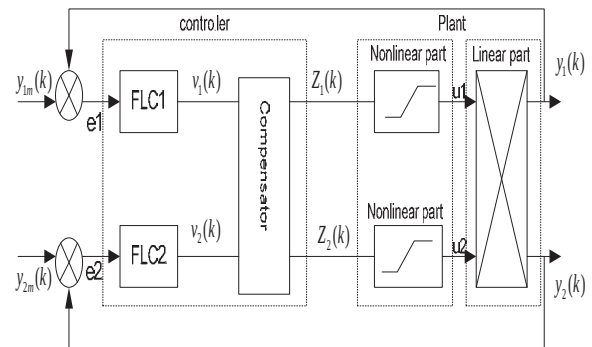


Figure 1. Diagram schematic of the control



The problem includes the choice of the input and output variables of the fuzzy control rules, the generation of the rules and placing the membership functions. The fuzzy logic control law $u_i(k)$ considered in this paper is the form:

$$\Delta u_i(k) = f(e_i(k), \Delta e_i(k)), \quad i = 1, \dots, n \quad (20)$$

Where $e_i(k) = y_i(k) - y_{im}(k)$, $\Delta e_i(k) = e_i(k) - e_i(k-1)$ and $f: R \times R \rightarrow R$ is a mapping that realises the control strategy desired by membership functions and rules of the controller. Each input of the fuzzy logic controller (FLC) has three fuzzy subsets: negative (N), zero (Z) and positive (P).

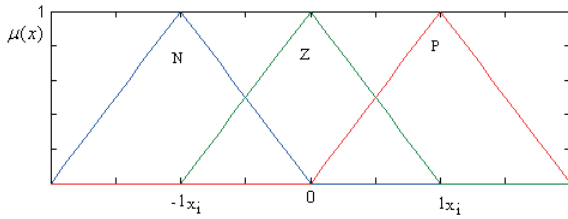


Figure 2. Membership functions for fuzzy sets of errors and change errors

Where $x_i = \{e_i, \Delta e_i\}$ and $l_{x_i} = \{l_{e_i}, l_{\Delta e_i}\}$, parameters l_{e_i} and $l_{\Delta e_i}$ define the widths of the membership functions as shown in figure 2.

The output of the FLC is the change of the control signal $\Delta u_i(k)$. The final control signal is calculated by:

$$u_i(k) = u_i(k-1) + \Delta u_i(k) \quad (21)$$

For each FLC, the form of l^{th} rule as the form:

if $e_1(k)$ is N and $\Delta e_1(k)$ is P then $\Delta u_i^l(k)$ is A^l

With $A = [-p_{VB}, -p_B, -p_M, -p_S, 0, p_S, p_M, p_B, p_{VB}]$, where S signifies small, M medium, B big and VB very big and each element of the matrix A is the place of the singleton which is used as membership functions.

Weighted sum is used as a defuzzification method which can be presented in a general form as:

$$\Delta u_i(k) = \varepsilon \sum_{j=1}^K P_j \mu_j \quad (22)$$

Where μ_j is the value of the membership function of the j^{th} fuzzy subset of $\Delta u_i(k)$, K is the number of the fuzzy subset of $\Delta u_i(k)$, P_j is the place of the j^{th} subset in universe of discourse of $\Delta u_i(k)$ and ε is the scaling factor or fine tuning parameter.

The control law $v_i(k)$ is given by:

$$\begin{aligned} v_i(k) &= u_i(k) + u_{ic}(k) \\ &= u_i(k) + C_i^T T(k) \end{aligned} \quad (23)$$

Equation (19) can be written as:

$$R_i(q^{-1})u_i(k) + G_i(q^{-1})y_i(k) + \quad (24)$$

$$R_i(q^{-1})C_i^T T(k) = A_{im}(q^{-1})y_{im}(k)$$

with

$$C_i^T = q^{d_{ii}} \frac{h_i^T B_{ii}^*(1)}{B_{ii}^*(q^{-1})} \quad (25)$$

The polynomial $R_i(q^{-1})$ and $G_i(q^{-1})$ are given by:

$$R_i(q^{-1})A_i(q^{-1}) + q^{-d_{ii}}G_i(q^{-1})B_{ii}^*(q^{-1}) = A_{im}(q^{-1}) \quad (26)$$

The closed loop equation is given as follows:

Operating on (12) by $R_i(q^{-1})$ gives:

$$R_i(q^{-1})A_i(q^{-1})y_i(k) = q^{-d_{ii}}R_i(q^{-1})B_{ii}^*(q^{-1})u_i(k) + \quad (27)$$

$$R_i(q^{-1})h_i^T T(k) + R_i(q^{-1})\sigma_i(k)$$

Combining equations (24) and (27), we obtain:

$$\begin{aligned} [R_i(q^{-1})A_i(q^{-1}) + q^{-d_{ii}}G_i(q^{-1})B_{ii}^*(q^{-1})]y_i(k) \\ + R_i(q^{-1})[q^{-d_{ii}}B_{ii}^*(q^{-1})C_i' - h_i']T(k) = \end{aligned} \quad (28)$$

$$q^{-d_{ii}}B_{ii}^*(q^{-1})A_{im}(q^{-1})y_{im}(k) + R_i(q^{-1})\sigma_i(k)$$

Combining equations (12), (24), (25) and (26) gives:

$$A_{im}(q^{-1})[y_i(k) - y_{im}(k)] = R_i(q^{-1})\sigma_i(k) \quad (29)$$

It is easily proved in [13] that the boundedness of the input-output signal will follow from a sufficient

smallness of the ratio $\frac{|\sigma_i(k)|}{\eta_i(k)} \leq \omega_i$. It is noticing that,

this assumption is less restrictive than the standard one imposed on equation (7): $\frac{H_i(k)}{\eta_i(k)}$ is sufficiently small in

the mean. [13, 14].

Theorem: For the system (1), if the control law (24) is used under assumptions A.1-3, then the closed-loop system is stable in the sense that:

1. The output and all inputs are bounded for all the time.
2. The output tracking error: $\lim_{k \rightarrow \infty} \{y_i(k) - y_{im}(k)\} = 0$ if $\sigma_i(k) = 0$.

The proof may be carried out along the same lines as in [16] and is then omitted.

5. Simulation results

Consider the two-input two-output (TITO) system having a polynomial nonlinearity described as follows [18]:

$$A(q^{-1})y(k) = B_d(q^{-1})Z(k) + \xi(k)$$

Where q^{-1} is the back shift operator.



$y(k) \in R^2$, $Z(k) \in R^2$ and $\xi(k) \in R^2$ are respectively the output, nonlinear input and the disturbances vector of the TITO system. The nonlinearity is assumed to be represented as follows:

$$Z_i(k) = f_{i0} + f_{i1}u_i(k) + \dots + f_{ip}u_i^{p_i-1}(k)$$

The TITO system can be decomposed into two MISO nonlinear subsystems as follows:

$$\begin{aligned} A_1(q^{-1})y_1(k) &= q^{-d_{11}}B_{11}(q^{-1})Z_1(k) + \\ &\quad q^{-d_{12}}B_{12}(q^{-1})Z_2(k) + \xi_1(k) \\ A_2(q^{-1})y_2(k) &= q^{-d_{22}}B_{22}(q^{-1})Z_2(k) + \\ &\quad q^{-d_{21}}B_{21}(q^{-1})Z_1(k) + \xi_2(k) \end{aligned}$$

With

$$\begin{aligned} A(q^{-1}) &= \begin{bmatrix} 1 + 0.35q^{-1} + 0.15q^{-2} \\ 1 + 0.72q^{-1} + 0.05q^{-2} \end{bmatrix} \\ B_d(q^{-1}) &= \begin{bmatrix} 1 + 0.85q^{-1} & 2 + 1.25q^{-1} \\ 1.12 + 0.65q^{-1} & 1.65 + 0.23q^{-1} \end{bmatrix} \end{aligned}$$

$$d_{11} = 1, d_{12} = 2, d_{21} = 2 \text{ and } d_{22} = 1.$$

$$Z_1(k) = 0.5u_1(k) + 0.25u_1^2(k)$$

$$Z_2(k) = u_2(k) + 0.83u_2^2(k)$$

$$\xi_1(k) = 0.1 \text{rand}(1.600)$$

$$\xi_2(k) = 0.12 \text{rand}(1.600)$$

The inputs are $u_1(k)$ and $u_2(k)$. The outputs are $y_1(k)$ and $y_2(k)$ with $\xi_1(k)$ and $\xi_2(k)$ representing disturbances.

After some manipulation, we obtain:

$$A_1(q^{-1})y_1(k) = q^{-d_{11}}B_{11}^*(q^{-1})u_1(k) + H_1(k)$$

$$\text{with } q^{-d_{11}}B_{11}^*(q^{-1}) = 0.5q^{-1} + 0.425q^{-2}$$

$$A_2(q^{-1})y_2(k) = q^{-d_{22}}B_{22}^*(q^{-1})u_2(k) + H_2(k)$$

$$\text{with } q^{-d_{22}}B_{22}^*(q^{-1}) = 1.65q^{-1} + 0.25q^{-2}$$

$H_1(k)$ and $H_2(k)$ are the function contains the nonlinear terms, cross-coupling and disturbances.

To synthesise the FLC, the widths of the membership functions are: $L = [l_{e_1}, l_{\Delta e_1}, l_{e_2}, l_{\Delta e_2}] = [1, 1, 6.4, 6]$.

Let $\varepsilon = 0.1$.

The final values of L and ε are obtained a few simulation tests.

The places of the membership functions for all the consequence is chosen as:

$$P = \begin{bmatrix} 14.35 & -3.42 & -3.20 & 10.5 & 0 \\ & -5.3 & 10.47 & -1.35 & -14.35 \\ 14.35 & -3.42 & -0.20 & 5.7 & 0 \\ & -1.39 & 3.42 & -0.33 & -14.35 \end{bmatrix}$$

The number of rules for this example is 9 rules.

It can be seen from Figures 3 and 4 that the performance of the decoupling via orthogonal polynomials and the proposed FLC is demonstrated and the tracking is obtained after small transient time.

The fine tuning parameter ε was increased while the interactions were small enough.

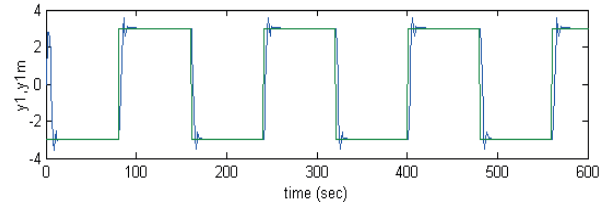


Figure 3. The response of $y_1(k)$ and $y_{1m}(k)$

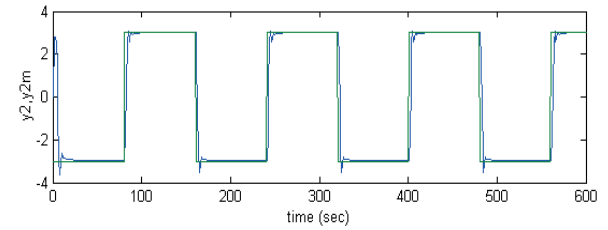


Figure 4. The response of $y_2(k)$ and $y_{2m}(k)$

The corresponding control laws are given on figures 5 and 6.

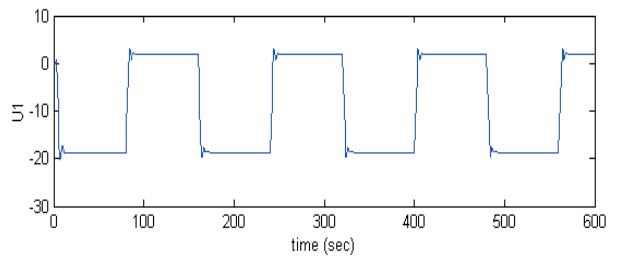


Figure 5. The behaviour of the control signal $u_1(k)$

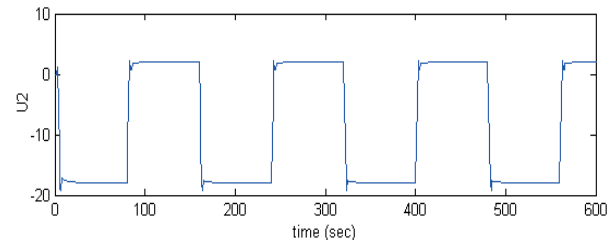


Figure 6. The behaviour of the control signal $u_2(k)$

6. Conclusion

The approach presents a solution to the problem of robust control of MIMO non linear systems. For each MISO subsystem a local fuzzy PI control is given. The effect of nonlinear dynamics, disturbances and cross-coupling is compensated by projecting it on Chebyshev polynomials.



This leads to both solving the nonlinearity problem and to releasing the decoupling through the control synthesis. The proposed control allows the direct implementation for a large class of MIMO systems. The simulation results show the effectiveness of the fuzzy adaptive PI controller.

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SWT and Statistical Hypothesis Testing for Power Inverter FDI

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** in memoriam*

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Abstract

In this paper, a monitoring approach for fault detection in power system drives is presented. Due to its critical position in the power train the diagnosis of the two level three phase voltage inverter is of a great concern. Fault scenarios with single open-switch are considered because they are the most likely to occur. Several signals are analysed simultaneously in order to perform the diagnosis. The fault occurrence is revealed by a change in the mean value of a subset of the analysed signals. The diagnosis is realised in three steps. In the first step, signals are filtered using the SWT performed with the DB4 wavelet to extract the detail and approximation coefficients up to level 6. In the second step the approximation at level 6 is examined to detect changes in the mean. This is achieved with statistical hypothesis techniques. A Neyman-Pearson change in the mean detection test is used. Then at the third step, a signature table allows to isolate the faulty switch. The whole diagnostic procedure can perform on line because of its low computational cost. Real data recorded from a benchmark feed the proposed diagnostic tool. The results Presented here confirm the effectiveness of the proposed methodology.

Keywords: *Fault Detection and Isolation (FDI), Stationary Wavelet Transform (SWT), Neyman-Pearson detection test, power converter, hypothesis testing.*

1. Introduction

Fault Detection and Isolation (FDI) is an essential aspect of process engineering. It is important not only from a safety viewpoint but also for the maintenance politics. FDI has received a considerable attention from industry and research communities because of the economic and safety impacts involved. There is however a number of practical challenges in designing robust FDI systems. Among the difficulties are the complexity of process, the lack of adequate models, the incomplete and uncertain

data, the partial knowledge on the system, the effort and expertise required to develop and maintain the FDI systems.

Signal processing is a well known tool to achieve FDI. It is used to analyze directly the signals measured online, avoiding system modelling. Its main drawback is that a change in some signal feature, e.g. the signal mean or the frequency contents, must be distinctive for each fault that may occur on the system.

Detection tests that aim at detecting a change in the mean or the standard deviation of a signal are now very common [1, 2] techniques to perform diagnosis. Frequency representations are particularly useful for anomaly detection as for example in induction machines and drive systems. Indeed, extra frequency contents may appear under the influence of a particular fault. For instance, [3] deeply studies faults in a three-phase induction machine. The spectral analysis of electric and electromagnetic signals shows that mechanical abnormalities such as broken rotor bars generate characteristic frequency contents in the signals.

Unfortunately, the Fourier Transform is unable to accurately analyse and represent a signal with non periodic features, such as transient ones. To study non stationary signals, time-frequency methods replace traditional spectral analysis [4, 5]. The Short Time Fourier Transform interpretation is close to a local Fast Fourier Transform analysis. The signal to analyse is multiplied by a sliding window with finite duration. Thus the spectrum is computed in real time and the variation of its contents is used to detect faults.

The main drawback of STFT method is due to the constant time-frequency resolution, according to the Heisenberg-Gabor uncertainty principle. Indeed, there is a trade-off between time and frequency resolutions because an accurate time resolution requires a "short" analysis window while an accurate frequency resolution involves a "long" analysis window. However, in the context of diagnosis, this long analysis window introduces extra detection delays.



In order to obtain variable time and frequency resolutions with constant product, the Wavelet Transform (WT) has been introduced. Moreover, Wavelet analysis does not require stationarity hypothesis and it is well adapted to the analysis of signals with changes over the time. Multiresolution analysis (MRA) has been investigated for monitoring and diagnosis in various industrial areas [6, 7]. It is particularly suitable when no model of the physical phenomenon is available. In power converters, commutation failures and single phase short circuits at the AC side have been studied with such a tool. It has been shown that these faults produce time varying transients. [8] reports the use of the MRA to exhibit faults in power system drive.

The Stationary Wavelet Transform (SWT) may be preferred to the Discrete Wavelet Transform (DWT) since it does not include any downsampling that may introduce a delay in the fault detection. The SWT fills in the gaps between the coefficients in any particular decimated DWT.

In order to perform the FDI, the deviation of features from normal situation must be detected. Statistical hypothesis techniques can be applied in order to exhibit changes. From a practical viewpoint, the FDI framework usually implements two hypotheses, namely “unfaulty situation, or at least undetectable deviation from the normal behaviour” and “deviation from the normal behaviour”. It was shown in [9] that an optimal alarm system is fundamentally based upon a likelihood ratio criterion via the Neyman-Pearson lemma. The benefit from the trade-off between false alarm probability and missed detection probability is important to make decision rules. This allows to design an optimal alarm system that will elicit the smallest miss detection rate for a fixed false alarm rate.

In this paper, the diagnosis of a power converter associated with an induction motor is considered because of its vital position in the power train. Faults in the converter may induce dramatic damages on the power system. Therefore, the development of a real-time diagnostic algorithm is of great interest. In order to study various fault scenarios, a simulator has been developed considering one single open-switch because these faults are the most likely to occur. Several signals must be analysed simultaneously in order to perform the fault isolation.

The remainder of the paper is organized as follows: Section (2) focuses on the SWT used as a preprocessing tool to feed the detection and isolation phases of the FDI procedure. Section (3) emphasizes on the statistical hypothesis test that will be applied on the approximation coefficients during the detection step of the diagnostic procedure. Section (4) gives a brief description of the benchmark with its simulator and presents the diagnostic tool steps. The results achieved with the proposed method on real data are discussed in section (5).

2. Stationary Wavelet Transform Theory

A detection method based upon the MRA has been previously proposed in the context of power inverter diagnosis [10]. However, the main drawback of this non-redundant transform [14] is its non-invariance in time/space, i.e. the coefficients of a delayed signal are not

a time-shifted version of those of the original signal. It is well admitted that time invariance is very important for diagnostic purpose and the DWT algorithm does not meet this requirement [15]. The SWT was introduced to make the wavelet decomposition time invariant. Among its appealing properties, the SWT improves the discontinuity detection capabilities of the DWT, as stated in [18]. However, the computational cost of the SWT compared with its MRA counterpart is significant, respectively $O(N \log N)$ and $O(N)$, where N is the length of the signal. Thus, it must be taken into account when online implementation of the FDI procedure is considered [17].

In the present paper, the SWT is used instead of the MRA for the detection of changes in the “low” frequency part of the signal while classical uses of the SWT in the diagnostic framework deal with the “high” frequency part of the signal [16]. The SWT method is used here for its filtering capability. It can be described as follows: at each level, when the lowpass and highpass filters are applied to the data, the two new sequences have the same length as the original sequence. To do this, the original data is not decimated. However, the filters at each level are modified by padding them out with zeros [11]. Suppose that a function $f(x)$ is projected at each step j on the subset $V_j(\dots \subset V_3 \subset V_2 \subset V_1 \subset V_0)$. This projection is defined by the scalar product $a_{j,k}$ of with the scaling function $\phi(x)$ which is dilated and translated.

$$a_{j,k} = \langle f(x), \phi_{j,k}(x) \rangle \quad (1)$$

$$\phi_{j,k}(x) = 2^{-j} \phi(2^{-j}x - k) \quad (2)$$

where $\phi(x)$ is the scaling function, which is a low-pass filter. $a_{j,k}$ is also called a discrete approximation at the resolution 2^j . If $\varphi(x)$ is the wavelet function, the wavelet coefficients are obtained by

$$d_{j,k} = \langle f(x), 2^{-j} \varphi(2^{-j}x - k) \rangle \quad (3)$$

$d_{j,k}$ is called the discrete detail signal at the resolution 2^j . As the scaling function $\phi(x)$ has the following property:

$$\frac{1}{2} \phi\left(\frac{x}{2}\right) = \sum_n h(n) \phi(x - n)$$

$a_{j+1,k}$ can be obtained by direct computation from $a_{j,n}$

$$a_{j+1,k} = \sum_n h(n - 2k) a_{j,n} \quad (4)$$

Moreover

$$\frac{1}{2} \varphi\left(\frac{x}{2}\right) = \sum_n g(n) \phi(x - n)$$

The scalar products $\langle f(x), 2^{-(j+1)} \varphi(2^{-(j+1)}x - k) \rangle$ are computed with

$$d_{j+1,k} = \sum_n g(n - 2k) a_{j,n} \quad (5)$$

Equations (4) and (5) are the multiresolution algorithm of the traditional DWT. In this transform, a downsampling step is used to perform the transformation. That is, one point out of two is kept during the computation at one particular level. Therefore, the whole length of the function $f(x)$ will reduce by half at each level.



However, for stationary or redundant transform, instead of downsampling, an upsampling procedure is carried out before performing filter convolution at each scale. Thus, the approximation coefficient $a_{j+1,k}$ is obtained by:

$$a_{j+1,k} = \sum_n h_j(n-k)a_{j,n} \quad (6)$$

and the discrete wavelet coefficient is

$$d_{j+1,k} = \sum_n g_j(n-k)a_{j,n} \quad (7)$$

where h_j and g_j are the h and g filters with $2j-1$ zeros between each filter coefficient. The redundancy of this transform facilitates the identification of salient features in a signal.

3. Neyman Pearson Approach

When FDI is performed using signal processing techniques, the fault occurrence is supposed to change some of the signal characteristics. This change is exhibited with a fault indicator (the symptom) which can be built from a decision test established according to, for instance, statistical hypothesis tests.

In binary hypothesis testing problem [1], the two hypotheses are denoted by H_0 and H_1 . The hypothesis H_0 is known as the null hypothesis. This is because it usually represents the absence of fault (or at least undetectable fault). The hypothesis H_1 is known as the alternative hypothesis and it usually represents the presence of fault. P_1 is the probability distribution function of the signal under hypothesis H_1 and P_0 is the probability distribution function of the signal under hypothesis H_0 . Two types of errors are defined. The first kind of error occurs when H_0 is true and H_1 is decided. This error is the "Type I error". The probability corresponding to this type of error is known as the false alarm probability or false alarm rate α . The second kind of error occurs when H_0 is decided while H_1 is true. This error is the "Type II error". The probability corresponding to Type II error is named the miss detection probability β .

The design of a hypothesis test involves a trade-off between α and β . The goal of detection theory is to choose an appropriate test or decision rule that is optimal in some sense: either it should minimize the probability of error, or it should maximize the probability of detection, or it should minimize the conditional risk, etc. Therefore, depending on the goal, there are different ways of customising the decision rule. The four situations are summarised in table 1 where:

$$\alpha = \Pr(H_1|H_0) \quad \text{and} \quad \beta = \Pr(H_0|H_1)$$

	H_0 true	H_1 true
Decided H_0	$D_{00} = 1 - \alpha$	$D_{01} = \beta$
Decided H_1	$D_{10} = \alpha$	$D_{11} = 1 - \beta$

Table 1. Retained decision

In the literature, various detection tests can be found such as Bayes, Minimax, Neyman-Pearson... In the present work, the Neyman-Pearson test has been applied. This test maximises the detection probability $(1-\beta)$ if one accepts a certain probability of false alarm α . Fig. 1 shows the probability distributions P_0 and P_1 and the

associated decision that corresponds to the threshold value λ which is deduced from α . It also shows the false alarm probability and the miss detection probability β . It clearly exhibits the trade-off between α and β .

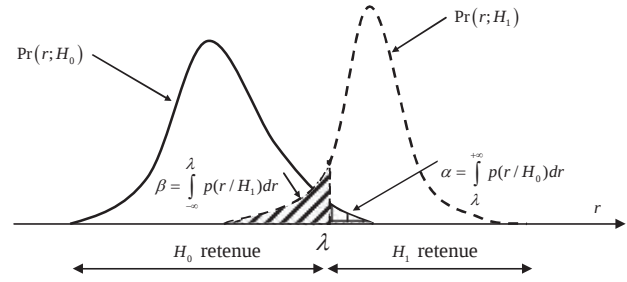


Fig. 1 False alarm and miss detection values for a certain threshold

The following test is now considered:

$$H_0 : r = \mu_0 + \nu \quad (8)$$

$$H_1 : r = \mu_1 + \nu$$

Therefore (8) corresponds to a change in the mean detection test, with μ_0 and μ_1 the means under hypotheses H_0 and H_1 respectively. ν is a white noise. Let $P_1 = \Pr(r|H_1)$ and $P_0 = \Pr(r|H_0)$. The Neyman-Pearson lemma involves the so-called likelihood ratio $V(r)$ defined by [1]:

$$V(r) = \frac{\Pr(r|H_1)}{\Pr(r|H_0)} \underset{H_0}{\overset{H_1}{>}} \eta$$

If $V(r)$ passes a threshold η , the choice is H_1 , otherwise the choice is H_0 . The threshold is computed under the constraint on the false alarm probability:

$$\alpha = \int_{\lambda}^{+\infty} \Pr(r_1|H_0) dr$$

The usual way is to consider the logarithm of the likelihood ratio, named the log-likelihood ratio:

$$W(r) = \ln(V(r)) \underset{H_0}{\overset{H_1}{>}} \ln(\eta) \quad (9)$$

Under the Gaussian noise hypothesis (ν is $N(0, \sigma^2)$), it can be shown that:

$$\lambda = \frac{\sigma^2}{\mu_1 - \mu_0} \ln(\eta) + \frac{\mu_1 - \mu_0}{2} \quad (10)$$

In this work, the case of a multidimensional signal is considered. The hypotheses become:

$$H_0 : r_n = \mu_0 + \nu'_n; \quad n=1:N \quad (11)$$

$$H_1 : r_n = \mu_1 + \nu'_n; \quad n=1:N$$

The Gaussian white noise independent identically distributed (iid) case is obvious and it is not presented in this paper. From now on, the noise sequence $\{\nu'_n\}$ is supposed correlated with covariance matrix C . This situation may arise in linear filtering, r_n being computed with:

$$r_n = \mathbf{h}^T \mathbf{x}_n \quad (12)$$



where $\mathbf{h} = [h_1, \dots, h_L]^T \in \mathfrak{R}^L$ is the filter impulse response and $\mathbf{x}_n = [x_{n-L+1}, \dots, x_{n-1}, x_n]^T \in \mathfrak{R}^L$. Consider that the raw data x_n satisfies:

$$x_n = \tilde{x}_n + v_n \quad (13)$$

$\tilde{x}_n$ is the deterministic part of the sample x_n . v_n is supposed to be a zero mean Gaussian white noise $N(0, \sigma^2)$ and the sequence $\{v_n\}$ are iid variables. Therefore:

$$\begin{aligned} r_n &= \mathbf{h}^T \tilde{\mathbf{x}}_n + \mathbf{h}^T \mathbf{v}_n \\ &= \tilde{r}_n + v'_n \end{aligned} \quad (14)$$

and

$$\tilde{\mathbf{x}}_n = [\tilde{x}_{n-L+1}, \dots, \tilde{x}_n]^T \in \mathfrak{R}^L. \quad (15)$$

The elements of $\mathbf{v}'_n = [v'_{n-L+1}, \dots, v'_n]^T \in \mathfrak{R}^L$ are correlated noises with covariance matrix equal to:

$$C = \sigma^2 A A^T \quad (16)$$

where

$$A = \begin{bmatrix} h_1 & h_2 & \dots & h_L & 0 & \dots & 0 \\ 0 & h_1 & h_2 & \dots & h_L & 0 & 0 \\ \vdots & \vdots & & & & \ddots & \vdots \\ 0 & 0 & 0 & h_1 & h_2 & \dots & h_L \end{bmatrix} \in \mathfrak{R}^{N \times (N+L-1)} \quad (17)$$

Without lack of generality, consider $\mu_0 = 0$ in (11). The hypotheses become:

$$H_0 : r_n = v'_n; \quad n = 1 : N$$

$$H_1 : r_n = \mu_1 + v'_n; \quad n = 1 : N$$

Define $\mathbf{r} = [r_1, \dots, r_N]^T \in \mathfrak{R}^N$ and $\mathbf{m}_1 = [\mu_1, \dots, \mu_1]^T \in \mathfrak{R}^N$.

Then, the probability functions under hypotheses H_0 and H_1 are respectively:

$$P_0 = \Pr(r|H_0) = \frac{1}{\sqrt{(2\pi)^N \det(C)}} e^{-\frac{1}{2}(\mathbf{r}^T C^{-1} \mathbf{r})} \quad (18)$$

$$P_1 = \Pr(r|H_1) = \frac{1}{\sqrt{(2\pi)^N \det(C)}} e^{-\frac{1}{2}((\mathbf{r}-\mathbf{m}_1)^T C^{-1} (\mathbf{r}-\mathbf{m}_1))} \quad (19)$$

The likelihood ratio becomes:

$$V(r) = \frac{\frac{1}{\sqrt{(2\pi)^N \det(C)}} e^{-\frac{1}{2}((\mathbf{r}-\mathbf{m}_1)^T C^{-1} (\mathbf{r}-\mathbf{m}_1))}}{\frac{1}{\sqrt{(2\pi)^N \det(C)}} e^{-\frac{1}{2}(\mathbf{r}^T C^{-1} \mathbf{r})}}$$

$$V(r) = e^{-\frac{1}{2}(-2\mathbf{r}^T C^{-1} \mathbf{m}_1 + \mathbf{m}_1^T C^{-1} \mathbf{m}_1)} \underset{H_0}{\overset{H_1}{>}} \eta$$

and its logarithm (9) is:

$$\begin{aligned} \ln(V(r)) &= \frac{-1}{2}(-2\mathbf{r}^T C^{-1} \mathbf{m}_1 + \mathbf{m}_1^T C^{-1} \mathbf{m}_1) \underset{H_0}{\overset{H_1}{>}} \ln(\eta) \\ \Leftrightarrow \mathbf{r}^T C^{-1} \mathbf{m}_1 &\underset{H_0}{\overset{H_1}{>}} \ln(\eta) + \frac{\mathbf{m}_1^T C^{-1} \mathbf{m}_1}{2} \end{aligned} \quad (20)$$

As expected, the noise correlation must be taken into account and obviously, the threshold depends on matrix C. Note that (20) corresponds to the computation of a

weighted sum of the samples r_n while the iid case considers the unweighted sum $\sum_{n=1}^N r_n$.

Even if the computation of (20) is of no difficulty, a suboptimal solution is derived with the noise sequence $\{v'_n\}$ supposed to be uncorrelated. Consider the signal mean:

$$m = \frac{1}{N} \sum_{n=1}^N r_n \quad (21)$$

Thus, the hypotheses to be tested are:

$$H_0 : m = v'' \quad (22)$$

$$H_1 : m = \mu_1 + v''$$

where v'' is supposed to be a Gaussian white noise $N(0, \sigma'^2)$ and

$$\sigma'^2 = \frac{1}{N} \{ [1 \dots 1] C [1 \dots 1]^T \} \quad (23)$$

The likelihood ratio is given by:

$$\begin{aligned} V(m) &= \frac{\frac{1}{\sqrt{(2\pi)\sigma'^2}} e^{-\frac{1}{2\sigma'^2}(m-\mu_1)^2}}{\frac{1}{\sqrt{(2\pi)\sigma'^2}} e^{-\frac{1}{2\sigma'^2}(m)^2}} \\ \Leftrightarrow V(m) &= e^{-\frac{1}{2\sigma'^2}(-2m\mu_1 + \mu_1^2)} \underset{H_0}{\overset{H_1}{>}} \eta \end{aligned}$$

It follows that the detection test to be implemented is:

$$m \underset{H_0}{\overset{H_1}{>}} \lambda = \frac{\sigma'^2}{\mu_1} \ln(\eta) + \frac{\mu_1}{2} \quad (24)$$

This threshold is similar to the one computed in (10). This suboptimal detection test is implemented in the FDI procedure, the mean value of the deterministic part of the analysed signal over the window of length N being null.

4. Benchmark description

As stated above, the power converter reliability is extremely important in industrial applications. Thus a large amount of papers deal with the faulty mode behaviour of static converters, and with the fault tolerant control and protection of voltage source inverter systems. Some of them consider various fault modes of a voltage source PWM inverter for induction motor drive [12]. Others are interested in fault tolerant control of induction motor drive applications using analytical redundancy, providing solutions to the most frequent faults [13].

This paper deals with the fault detection and isolation problem of a voltage source PWM motor drive system. The one open switch fault case is considered, due to its frequent occurrence. A benchmark depicted in Fig. 2 has been developed. It consists of a three-phase inverter with IGBTs and anti-parallel diodes controlled by a DSpace DS1103 board inserted into a PC Pentium and an induction machine as a load. This mixed RISC/DSP digital controller, based on two microprocessors (power PC 604e-333MHz and TMS320F240-20MHz) and four high-resolution analog-to-digital (A/D) converters (0.8 μ s - 12 bits), provides an efficient platform for real-time floating point calculation. The controller board is fully



supported by the Matlab/Simulink environment. The main voltage supply is set to 200 V. The characteristics of the induction motor connected to the inverter are 1.8 kW, 380V, 4.5A and 1460 rpm.

The electrical drive has been tested under normal conditions and with an open circuit fault on the IGBT T4. Fig. 3 shows data recorded on the setup. The three phase stator currents and the control of IGBT T4 waveforms are shown. After the fault occurrence, the phase 2 current has a positive DC offset and a distorted waveform. This is due to the fact that during a commutation in leg 2, the load in phase 2 is connected to the DC positive voltage bus all the time through the defective IGBT. Negative DC offsets are then present in both phase 1 and 2 currents.

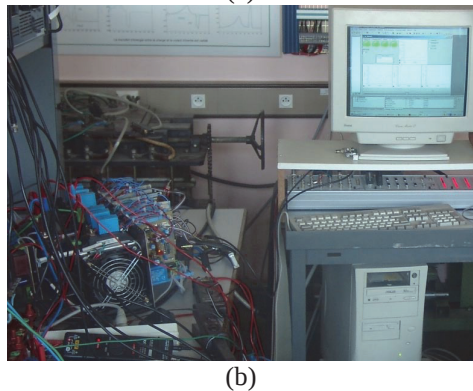
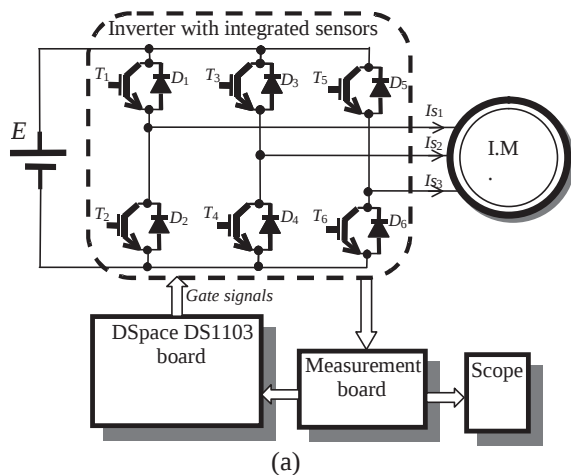


Fig. 2 (a) Configuration of the experimental setup, (b) experimental setup

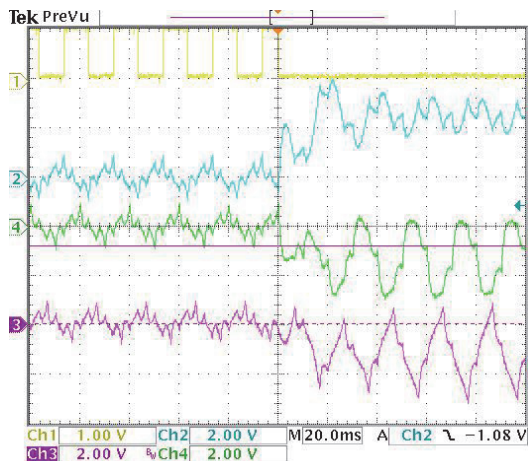


Fig. 3 Switch-on failure transient waveform: (1) control of IGBT T4, (2) phase 2 stator current, (4) Phase 3 stator current and (3) phase 1 stator current

As previously mentioned, faults are common phenomena in power inverter. They induce device destruction. Due to their cost constraints, a power inverter simulator using IGBT and anti-parallel diode detailed models was developed in the Matlab environment. The simulator has been set in the same operating conditions as the laboratory setup. Parameters are set equal to those of the benchmark. Fig. 4 exhibits the results obtained with the Matlab simulator. Simulation results have been compared successfully with measurements in steady state and transient conditions. It can be noticed the excellent agreement between the simulated and the recorded signals in both normal and faulty conditions [8].

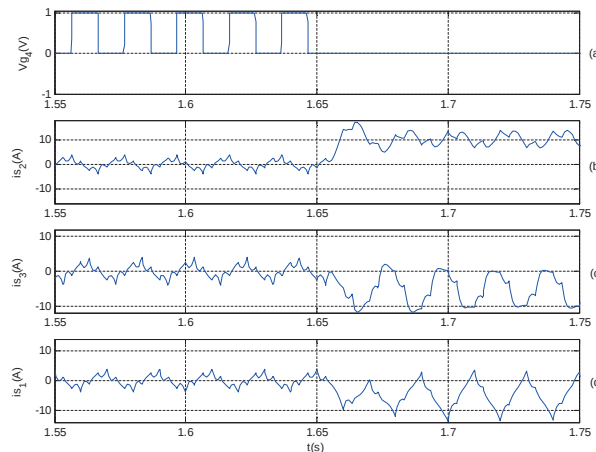


Fig. 4 Simulation results, zoom around the fault time occurrence, (a) control of IGBT T4, (b) Phase 2 stator current, (c) Phase 3 stator current and (d) Phase 1 stator current

5. Fault Detection and Isolation

The current signals in the three phases are analysed simultaneously. The output of the detection phase provides six symptoms that feed the isolation step. The fault isolation is performed with a signature table to isolate the faulty switch. Therefore the FDI procedure is organised in three steps Fig. 5:

- Preprocessing of the stator currents with the SWT to obtain the approximation a_6 at level 6;
- detection using Neyman Pearson tests (change in the mean detection tests);
- isolation with a signature table.

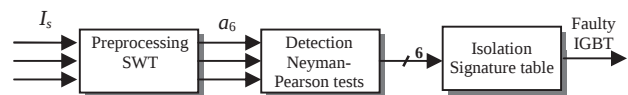


Fig. 5 FDI blocks diagram

In this section the three steps are presented. Then the results obtained with the data recorded from the benchmark are discussed.

5.1 SWT Preprocessing

The SWT is applied to the three-phase current signals using the Daubechies wavelet DB4. It decomposes the signal into approximation and detail coefficients. The signal analysis was performed up to the sixth level. Indeed, the approximation at level 6 emphasises the fault occurrence while high frequency contents are sufficiently filtered. A single transistor base drive open circuit of the



power converter is considered. An extensive investigation has been carried out for various faulty IGBTs. The mean value (on a time period) of the stator currents in steady state is zero. When the open circuit fault appears in the inverter transistor, the fault information affects several frequency bands Fig. 6.

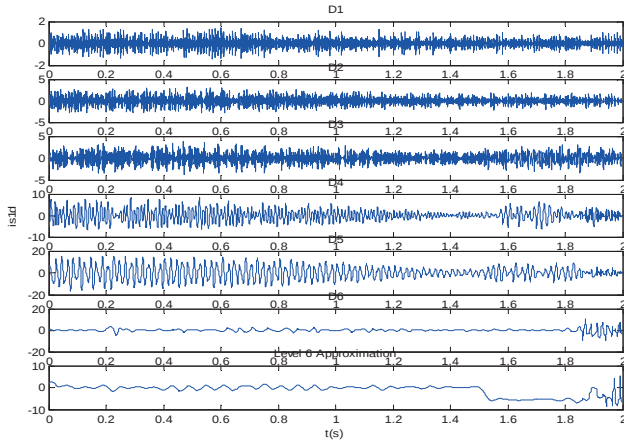


Fig. 6 Six levels of detail and approximation of phase 1 stator currents before and after open-circuit fault on IGBT T1

Spikes in the details at the sixth level and in the approximations can be seen. Their appearance time is directly correlated with the occurrence time of the fault. Moreover, this kind of fault introduces a non-zero DC offset that can be clearly observed in the approximation at level six. Fig 7 depicts a current and its approximation a_6 at level 6 when IGBT T1 fails. Fig. 8 shows that a larger DC offset appears in the faulty phase when compared to the other phases.

As a result of time domain studies, the faulty situation can be discriminated from the healthy one. Note that the 6th level approximation a_6 is greater than its DWT counterpart. Therefore, the gap (i.e. the mean change between the healthy and the faulty situations) is more easily detected in an automatic way. As a consequence, the detection of the mean change in a_6 is facilitated compared to the detection of the change in the current. However, a small unbalance of the load must not be confused with the faulty scenarios. Thus, the threshold has been computed in order not to introduce false alarms due to unbalance of the load.

5.2 Fault detection with Neyman Pearson

Approximation at level 6 is analysed with the Neyman-Pearson test presented in section 3. The false alarm probability α is set to 0.05 and the miss alarm probability is minimized.

The measured currents are sampled at $T_e = 0.2$ ms. These raw data are supposed to be modelled with:

$$i(t) = \tilde{i}(t) + v(t), \quad t = kT_e \quad (25)$$

where $\tilde{i}$ is a periodic deterministic signal, and v is a Gaussian white noise $N(0, \sigma^2)$. When a fault occurs, the current is supposed to be:

$$i(t) = \tilde{i}(t) + \mu_1 + v(t) \quad (26)$$

with μ_1 a constant.

The SWT is applied to i and the approximation a_6 is considered. Actually, the SWT is a linear transformation.

Therefore, with respect to (11), in normal condition, one gets:

$$a_6(t) = \mathbf{h}^T \mathbf{i} = \tilde{a}_6(t) + v'(t) \quad (27)$$

with $\tilde{a}_6$ a periodic deterministic signal and v' a Gaussian noise with zero mean and $\sigma^2 \mathbf{H}^T \mathbf{H}$ variance. Note that the sequence $\{v'\}$ is no longer independent. Its covariance matrix is given by (16).

The Neyman-Pearson test is performed on m (21):

$$\begin{aligned} m(t) &= \frac{1}{N} \sum_{i=1}^N a_6(t-i+1) \\ &= \frac{1}{N} \sum_{i=1}^N \tilde{a}_6(t-i+1) + \frac{1}{N} \sum_{i=1}^N v'(t-i+1) \end{aligned}$$

The window length N is chosen so that $\sum_{i=1}^N \tilde{a}_6(t-i+1) = 0$. The choice of $N = 200$ samples corresponds to 2 signal periods. Therefore, the hypotheses (19) are considered:

$$H_0 : m(t) = v''(t)$$

$$H_1 : m(t) = \mu_1 + v''(t)$$

where v'' is defined in (20).

For each stator current, two tests are implemented. Indeed, the sign of the mean change in a_6 is exploited to isolate the fault. The minimum mean change to be detected μ_1 is chosen so that the test is insensitive to a small unbalance of the load. The mean μ_1 is set to +20 or -20: the sign of the change will be used for isolation purpose. The test for the positive (negative) mean change is named $T_u(a_6 i_j)$ ($T_d(a_6 i_j)$), $j = 1:3$. The six Neyman-Pearson tests are then derived and their thresholds are computed in accordance with (21). For each test, the symptom has a binary value: “0” means that “hypothesis H_0 is kept” while “1” stands for “hypothesis H_1 is selected”.

Figs. 7 to 10 depict two scenarios which correspond to the open switch on IGBT T₁ and T₄ respectively. In the experimentations the load is balanced. Thus, the currents have the same amplitude in normal operating mode. Moreover, a transient phase can be seen from time 0.2s until time 1.35s.

Fig. 7 shows the stator current i_1 in phase 1 (blue curve) in the case of a fault on IGBT T₁ and its approximation at level 6 (red curve).

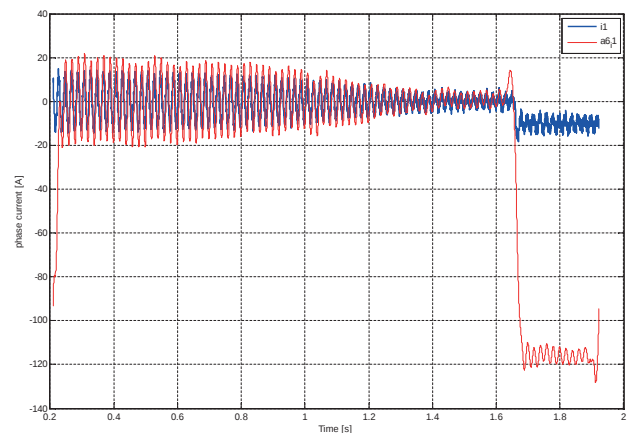


Fig. 7 Stator current in phase 1 (i_1) for the fault on IGBT T1 and its 6th approximation a_6



Fig. 8 depicts the three phase stator currents for the related fault scenario (top of the figure) and the six symptoms computed with the Neyman-Pearson tests. As expected, the detection tests $T_d(a6 i_1)$, $T_u(a6 i_2)$ and $T_u(a6 i_3)$ are sensitive to this fault.

Fig. 9 shows the stator current i_1 in phase 1 (blue curve) for the second fault scenario (faulty IGBT T4) and its approximation at level 6 (red curve).

Fig. 10 depicts the three phase stator currents (top of the figure) and the six symptoms obtained from the Neyman-Pearson tests. For this scenario, the detection tests $T_d(a6 i_1)$, $T_u(a6 i_2)$ and $T_d(a6 i_3)$ are sensitive to this fault. Note that the mean changes are different for both scenarios.

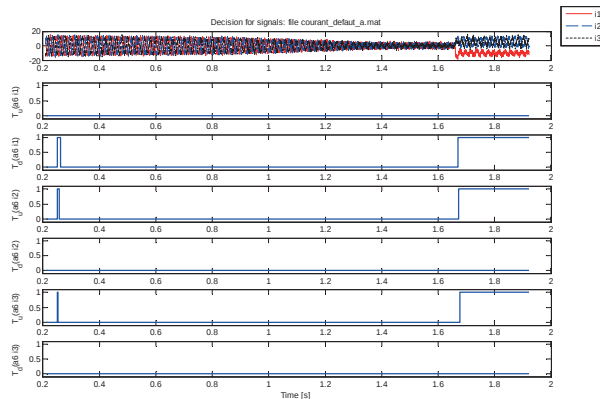


Fig. 8 Three phase stator currents for the fault on IGBT T_1 (top) and symptoms derived from the six Neyman-Pearson

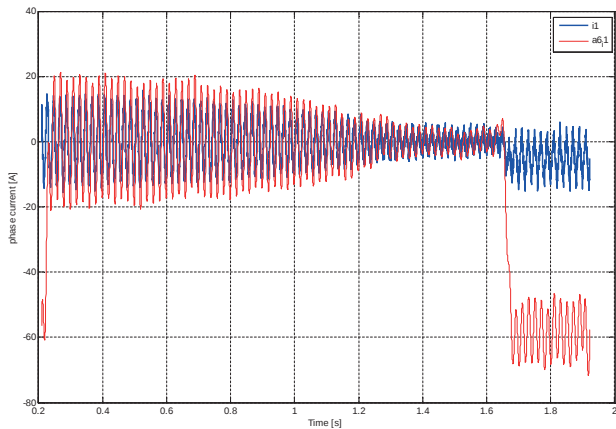


Fig. 9 Stator current in phase 1 (i_1) for the fault on IGBT T_4 and its 6th approximation a_6

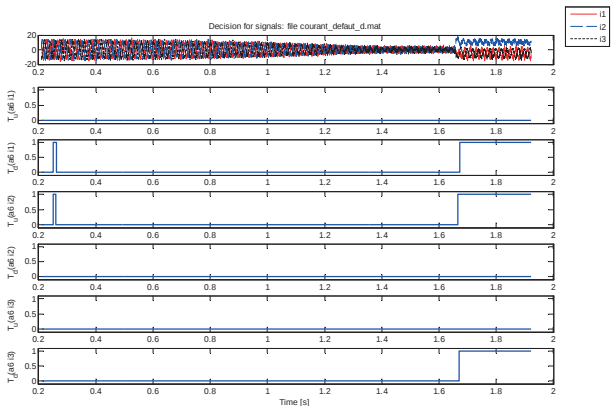


Fig. 10 Three phase stator currents for the fault on IGBT T_4 (top) and symptoms derived from the six Neyman-Pearson

5.3 Fault Isolation

The signature table was obtained with the simulator by making an exhaustive study of the various fault scenarios. The test results presented in section 5.2 make possible the isolation of the faulty switch. Indeed each combination of symptoms characterizes a fault. Thus the signature table is strongly locating. Table 2 gives the signature table. “0” means that hypothesis H_0 has been chosen for the considered test while “1” stands for H_1 has been kept. The isolation table has been used for the diagnosis of the setup plant. The fault isolation is based on a set of rules that are deduced from the isolation table.

Detection Test	Fault on IGBT ₁	Fault on IGBT ₂	Fault on IGBT ₃	Fault on IGBT ₄	Fault on IGBT ₅	Fault on IGBT ₆
$T_d(a6 i_1)$	0	1	1	0	1	0
$T_d(a6 i_2)$	1	0	0	1	0	1
$T_u(a6 i_2)$	1	0	0	1	1	0
$T_d(a6 i_2)$	0	1	1	0	0	1
$T_u(a6 i_3)$	1	0	1	0	0	1
$T_d(a6 i_3)$	0	1	0	1	1	0

Table 2. Signature table to isolate the faulty IGBT

Two scenarios have been considered on the real plant, namely, a fault on T_1 and a fault on T_4 . The results presented in Figs. 7 to 10 are coherent with the simulated ones. Therefore, it is concluded that the single open-switch fault for all the possible scenarios can be detected and that the faulty switch can be isolated.

6. Conclusion

This paper presents a fault detection and isolation technique based on three steps. The first one makes use of the Stationary Wavelet Transform (SWT) as a preprocessing tool. The second step implements a statistical hypothesis testing method to detect changes in the wavelet coefficients. A signature table is used to isolate the fault during the third step.

The SWT has been chosen as an alternative tool to the Discrete Wavelet Transform (DWT) mainly for its time-invariant property and its filtering capabilities. The approximation at a given level is analysed in the second step of the procedure. A Neyman-Pearson detection test is implemented to detect the fault occurrence. This test takes into account the statistical characteristics of the filtered noise. Indeed, this latter cannot be any more considered as an iid sequence. Several detection tests run in parallel, providing symptoms that are used with the signature table to isolate the fault.

The proposed FDI technique has been successfully applied to a power inverter associated with an induction machine which is widely spread in industrial applications and a basic part of electric vehicles. The power inverter is a critical element of the power train: a fault on a switch may induce serious damages on both the motor and the inverter itself. The single open switch fault case was considered for the various IGBTs. The three phase stator currents are simultaneously analysed with the SWT up to level 6. Each one of the three approximation coefficients is processed in parallel by two Neyman-Pearson



detection test that compute symptoms. Then the six symptoms allow the isolation of the faulty switch. A simulator has been developed in order to carry out extensive investigations of the various fault scenarios which have been exploited to define the signature table. Real data recorded on the plant have been used to validate the FDI algorithm.

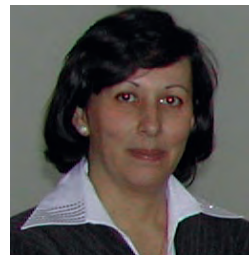
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On the Hysteresis Losses Investigation for Rotational Core of a SMPM in Time-stepped FEA

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Abstract

This paper deals with the modeling and computation of rotational hysteresis core losses in a rotating surface-mounted permanent magnet (SMPM) machine. Formulations of these losses taking into account the rotational behaviour of the magnetic field are developed. These formulations are used together with the time stepped finite element method to calculate the hysteresis core losses in the considered design according to the distribution of the rotating magnetic field. The originality of this work resides on the search of the optimum number of directions to be used for Vector Preisach model. This parameter has an interesting impact on the magnetic intensity value, and hence on the hysteresis loss value. So it seems significant to optimize this parameter by considering the two main criteria: (i) accurate estimation of the hysteresis iron losses, (ii) reasonable computation time.

In a first step, the effect of the number of directions on the hysteresis loss term has been investigated. In a second step, the hysteresis losses, considering the optimum number of directions and the developed formulations have been investigated. The calculated results are discussed.

Keywords: magnetic hysteresis losses, rotating electrical machines, time-stepped finite element analysis.

Nomenclature

f	frequency of the magnetic flux waveform
B_m	peak of the sinusoidal flux density
β	Steinmetz constant
k_h	hysteresis constant
λ	constant
θ	angle between B and the x-axis

α	angle between B and H
$ H_{e,ti} $	magnetic field intensity magnitude
$ B_{e,ti} $	induction magnitude
$H_{ex,ti}$	x- of $\mathbf{H}$
$H_{ey,ti}$	y- of $\mathbf{H}$
$B_{ex,ti}$	x- of $\mathbf{B}$
$B_{ey,ti}$	y- of $\mathbf{B}$
e	element
N	Total number of stepped FEA steps
p	Pair pole number
L	Stack length of the stator
A_i	Area of the finite element e
N_e	Total number of elements
$B_{m,i}$	Peak induction of the i^{th} element
$e_{\theta n}$	Unit vector along the direction θ_n
θ_n	Polar angle
$\Delta\theta$	Discretization angle
H_n^{SV}	Scalar Preisach corresponding to the direction n .
μ	Fixed point coefficient
R_{FP}	Fixed point residual
N_d	Number of directions

1. Introduction

A major part of the losses in electrical machines is the losses in the iron core. For an effective design of electric machines it is important to get accurate information about these losses. This is possible thanks to the knowledge of the magnetic field in the machine.

Because of the complicated geometry and operation modes of electric machines, the magnetic field variation is complex; hence a numerical technique is needed to solve this matter. In this way, the finite element analysis



(FEA) has been used to compute the magnetic field and thus the iron losses.

Generally, the iron losses are separated into hysteresis loss, classical eddy currents loss and excess loss. The present work deals with the computation of the first iron loss term.

In this paper an investigation of the hysteresis iron losses at no-load operation in a surface-mounted permanent magnet motor has been outlined.

Generally, in time-stepped FEA of electrical machines, single-valued curves are usually used for all materials comprised in the model. In this case, the irreversible material behavior in the iron cores completely ignored in the time stepped magnetic field equations, but the iron losses may readily be predicted a posteriori [1]–[3].

Another way to efficiently cure the negligence of the irreversible material behavior is the inclusion of an adequate hysteresis model in the FEA. In our case the used hysteresis model is the Vector Preisach Model, which has been incorporated in FEA in a reversed fashion. The vector Preisach model is defined as a superposition of scalar Preisach models regularly distributed along all possible angular directions [4] (a scalar model along each direction on the space). In practice, the number of direction is taken finite. This affects directly the value of the magnetic field H , and hence the hysteresis losses value. For these reasons, it has been the subject of deep investigations in the present work, in an attempt to reach the optimized number of directions to be adopted in the VPM used in this work. The considered design is a surface mounted permanent magnet motor (SMPM). In order to reduce the computation time, the calculation is made up in four regions of the considered topology.

2. Hysteresis loss formulations

According to the statistical theory [5], the average power loss per unit volume consists of the sum of the hysteresis, classical and excess loss contributions. In the present work, the hysteresis loss is the subject of investigation.

Many empirical and analytical formulae, dealing with the hysteresis loss modeling, depend on the nature of the flux density variation have been proposed in the literature. In a rotating machine, owing to the slotting and saturation effects, the magnetic flux density may be significantly distorted from the ideal sinusoidal variation. In addition, in some parts of the machine the magnetic flux density becomes rotational rather than alternating. Hence, a hysteresis model should take into consideration this rotational behavior of the magnetic flux density. For a long time, the hysteresis losses in rotating electric machines have been computed using alternating core loss model, in which the magnetic flux density is supposed to be purely alternating. This is due essentially to the lack of the data associated with the rotational core loss and the

lack of an appropriate model. According to the Steinmetz equation, the measurement and the calculation of the hysteresis losses are normally carried out with sinusoidal flux density of varying magnitude and frequency. These measurements and calculations are based on the following formula

$$P_h = k_h f B_m^\beta \quad (1)$$

f frequency of the magnetic flux waveform,

B_m peak of the sinusoidal flux density,

β Steinmetz constant

k_h hysteresis constant

For correction and modification, various models have been proposed [6]–[8],

$$P_h = k_h f B_m^{(a+b\beta)} \quad (2)$$

Where a and b are constant.

In a permanent magnet motor the magnetic flux density is far to be purely alternating, in certain region the flux density is purely rotational. So, a hysteresis loss model should take into account this rotational behavior of the flux density. For the modeling of rotational hysteresis loss, a basic Stoner–Wohlfarth model has been proposed in [9] and an empirical formula has been developed in [10]. Furthermore, the numerical implementation of such models is totally impractical.

$$P_h = P_{halt} + P_{hrot} = \lambda c(P_{halt}) \quad (3)$$

In [11] an estimation of the effect of rotational losses in an induction machine has been proposed. This approach estimates the iron losses by means three method: considering the losses provided by: purely alternating field, due to the orthogonal components of the flux density and by applying a correction factor based on experimental data to improve the rotational loss calculation. In this case it has been shown that the third method provides more accurate results than the other two. The lack of this method is the introduction of a correction coefficient depending on the flux density peak; this would considerably increases the computation time.

An investigation of the iron losses modeling has been reported in [12], in which the calculation of the iron loss in rotating electrical machines by using different iron loss models has been studied. The accuracy of each iron loss model was identified by the comparison between measured and calculated iron losses in the case of distorted elliptically flux density waveforms. In these models the calculation of the iron losses is based on the equivalent orthogonal alternating flux density components which.

An interesting iron loss model has been proposed in [13, 14]. In this model the hysteresis losses are expressed as follows:

$$p_h(t) = \frac{1}{T} \int_0^T H \frac{dB}{dt} dt = \frac{1}{T} \int_0^T \left(H_x \frac{dB_x}{dt} + H_y \frac{dB_y}{dt} \right) dt \quad (4)$$



Where T is the period of the magnetic field and $p_h(t)$ is the hysteresis power loss.

According to [14] the hysteresis loss in (4) can be expressed as follows:

$$p_h(t) = \frac{1}{T} \int_T |H| \frac{d|B|}{dt} \cos(\alpha) dt + \frac{1}{T} \int_T \frac{d\theta}{dt} (H \times B)_z dt \quad (5)$$

Where θ is the angle between the magnetic flux field B and the x-axis and α is the angle between B and H . The first term corresponds to the loss under purely alternating induction, in which $\frac{d\theta}{dt}$ is equal to zero and hence it can be expressed as follows:

$$p_{halt}(t) = \frac{1}{T} \int_T |H| \frac{d|B|}{dt} dt \quad (6)$$

Whereas the second term in (5) corresponds to the loss with purely alternating magnetic field ($\frac{dB}{dt} = 0$) and it can be expressed as follows:

$$p_{hrot}(t) = \frac{1}{T} \int_T \frac{d\theta}{dt} (H \times B)_z dt \quad (7)$$

This model will be adopted in the present work. And the total hysteresis power loss is obtained by summing the alternating and rotational components:

$$p_h = p_{alt} + p_{hrot}(t) = \frac{1}{T} \int_T |H| \frac{d|B|}{dt} dt + \frac{1}{T} \int_T \frac{d\theta}{dt} (H \times B)_z dt \quad (8)$$

By performing a 2-dimensional time-stepped FEA, the expression in (8) is transformed as follows:

$$p_h = \frac{1}{T} \int_T |H_{e,ti}| \frac{d|B_{e,ti}|}{dt} dt + \frac{1}{T} \int_T \frac{d\theta}{dt} (H_{ex,ti} B_{ey,ti} - H_{ey,ti} B_{ex,ti}) dt \quad (9)$$

Where $|H_{e,ti}|$, $|B_{e,ti}|$, $H_{ex,ti}$, $H_{ey,ti}$, $B_{ex,ti}$ and $B_{ey,ti}$ are respectively the magnetic field intensity and induction magnitudes, and the orthogonal components respectively of H and B . $t_i = 1, \dots, N$, N is the total number of stepped FEA steps. From the FEA the flux density and the field intensity for each element can be determined, taking into account the symmetry and periodicity in the structure. The analysis is limited to a half period. Then, the interval between consecutive time steps is:

$$\Delta t = t_i - t_{i-1} = \frac{T}{2N} \quad (10)$$

The magnetic hysteresis losses per element are derived from equation (9), then by summing these losses over the elements number the total hysteresis losses can be obtained as follows [17]:

$$p_h(t) = 2p \cdot f \cdot L \sum_{i=1}^{N_e} A_{e,i} \left(\frac{1}{T} \int_T |H_{e,ti}| \frac{d|B_{e,ti}|}{dt} dt + \frac{1}{T} \int_T \frac{d\theta}{dt} (H_{ex,ti} B_{ey,ti} - H_{ey,ti} B_{ex,ti}) dt \right) \quad (11)$$

Where p , L , A_i , N_e , $B_{m,i}$ are respectively the pair pole number, the stack length of the stator, the area of the finite element i and the total number of finite element mesh. The last equation has been coupled with the finite element analysis to calculate the hysteresis iron losses in a surface mounted permanent magnet motor. Owing to the symmetry and periodicity in the structure, the problem has been reduced to only one pole of the considered topology (figure 1). Doing so, the value of the obtained hysteresis losses is 34 W. Which, when compared to the one obtained in [15] (same value) shows that such a model is satisfactory.

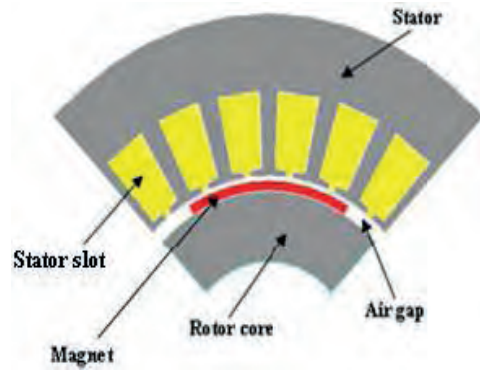


Figure 1: Fourth of the studied topology [16]

3. Hysteresis Modelling

In this paper, the hysteresis in ferromagnetic parts (stator core) is handled by means of the vector Preisach model, consisting of angularly distributed scalar models, which is applied in a reversed manner.

Considering a finite number of directions N_d equally spaced as illustrated in figure 2, H is computed as [18]:

$$H(t) = \frac{2}{\pi} \Delta\theta \sum_{n=1}^{N_d} H_n^{SV} (B \cdot e_{\theta_n}) e_{\theta_n} \quad (12)$$

e_{θ_n} is the unit vector along the direction specified by polar angle θ_n , $\Delta\theta$ is the discretization angle and H_n^{SV} is the scalar Preisach corresponding to the direction n . As presented in equation 12, the value of the magnetic field intensity H depends directly on the number of directions used in the vector Preisach model. This



influences explicitly the value of the hysteresis loss term given in equation 11.

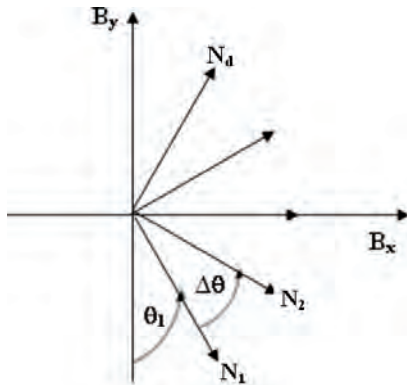


Figure 2: Finite number of direction.

For the integration of VPM in FEA an interface was needed. This interface is obtained by means of the fixed-point technique iterative procedure, in which the relationship between B and H is written as follows [17], [18]:

$$B = \mu H + R_{FP} \quad (13)$$

Where μ is the fixed-point coefficient and R_{FP} is a residual founded iteratively. Coupling equation (13) to appropriate Maxwell equations, the magnetic problem is formulated in terms of magnetic potential vector. The problem solution starts with an arbitrary given value of R_{FP} , providing the magnetic potential vector A, from which B is calculated and supplied as input of VPM giving the value of H. With B and H known, a new value of R_{FP} is calculated using formula (13). This value is again supplied in FEA equations. The procedure is repeated until the problem convergence is reached.

4. Effect of the Number of Directions

As discussed in the previous section, the number of directions used in the VPM has an increasing impact of the hysteresis losses term, within this intention a deep 2D FEA based investigations have been carried out in order to illustrate the effect of this parameter (number of direction) on the hysteresis losses. Doing so, the hysteresis losses in different elements have been computed for different directions. The obtained results are illustrated in figure 3.

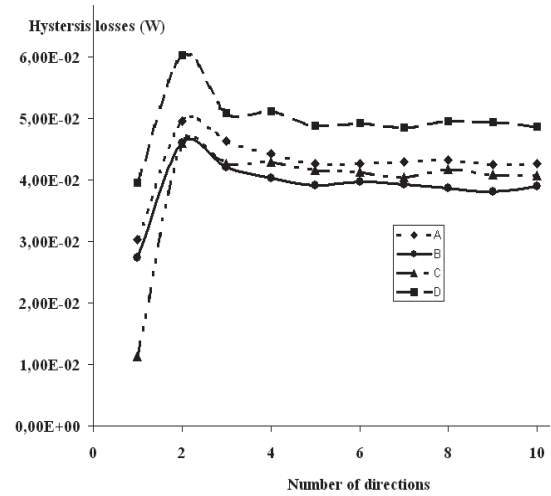


Figure 3: Hysteresis losses versus directions number

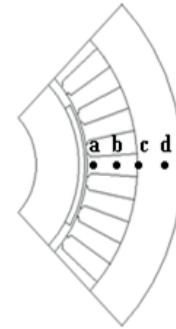


Figure 4: Points for which the hysteresis losses has been computed

As illustrated in figure 3 one can notice that the value of the hysteresis losses, for all regions, becomes slightly constant for a number of directions corresponding to 4. Then this parameter should be carefully chosen because low values lead to a bad estimation of the value of the iron losses and high values lead to the increase of the computation time.

In addition to the number of the VPM directions, there are many other parameters affecting directly the value of the hysteresis losses, such as the machine geometry essentially the pole number. For this reason, the effect of this parameter on the hysteresis losses has been investigated; the obtained results are illustrated in figure 5.



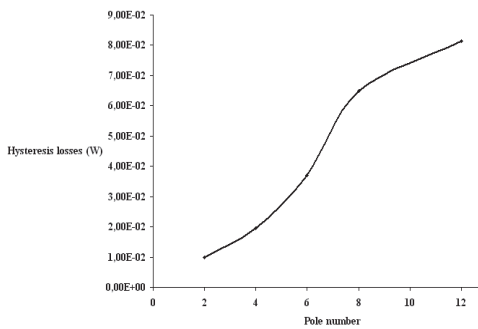


Figure 5: Hysteresis losses versus pole number

As illustrated in figure 5, one can clearly notice the increasing of the hysteresis losses with the pole number; this leads to the reduction of this parameter to obtain a low value of the hysteresis iron losses. However, the studied machine is a synchronous concept where the output power is proportional to the pole number, so a reduction of this parameter leads to the output power reduction. In this case a compromise should be taken. The flux lines plots corresponding to each pole number are illustrated in the follows figures.

Referring to these figures, we can notice on one hand the increase of the flux leakage and on the other the increase the field variation with the pole number yielding to a hysteresis losses increasing.

The effect of the other geometric parameter is implicitly considered in the stator teeth and yoke areas, which appears in the hysteresis losses expression in formula 11.

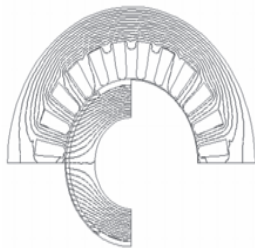


Figure 6: Flux lines plot: 2 poles

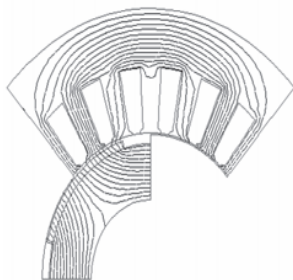


Figure 7: Flux lines plot: 4 poles

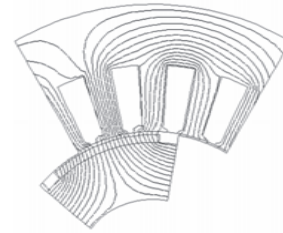


Figure 8: Flux lines plot: 6 poles

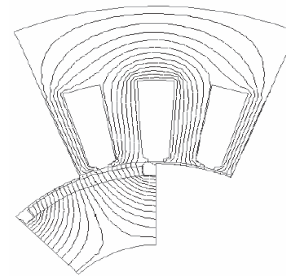


Figure 9: Flux lines plot: 8 poles

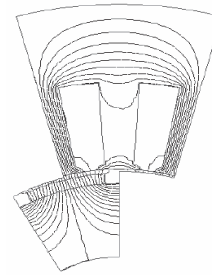


Figure 10: Flux lines plot: 12 poles

5. Conclusion

The hysteresis iron loss investigation in a SMPM machine at rotational field operation has been presented. For this reason, an analytical model was developed, and it has been used in conjunction with time-stepped FEA to estimate the hysteresis iron losses. This model provides reasonable accuracy, because it takes into consideration the rotational behavior of the magnetic field.

A brief study of the inclusion of the VPM in time stepped FEA has been also investigated. In this case a special attention is given to the effect of the number of directions used in the VPM in the hysteresis losses. The investigation of this parameter is required by the fact that in the expression of the hysteresis losses appears the magnetic field intensity H which depends clearly on the direction number. In this work, it was shown that this parameter has a crucial impact on the hysteresis losses value: for low values of this parameter the hysteresis losses value varies. Whereas, for high values the hysteresis loss becomes almost constant, furthermore the



computation time would also increase and hence a compromise should be taken. In this case a number of 4 directions were taken and would be adopted in order to accurately estimate the hysteresis losses. The effect of the pole number on the hysteresis losses has been investigated. In this case it has been shown the increasing of the hysteresis losses with this parameter; and hence the hysteresis losses reduction requires the reduction of this parameter. However, the studied machine is synchronous concept where the output power is proportional to the pole number, so a reduction of this parameter leads to the output power reduction. In this case a compromise should be taken

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A Qualitative Space Vector PWM Algorithm for a Five-level Neutral Point Clamped Inverter

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Abstract

In this paper a qualitative Space Vector Pulse Width Modulation (SVPWM) algorithm for five-level neutral point clamped inverter is proposed. With the proposed method, the three nearest compound vectors can be selected easily and dwelling time of each vector can be simply calculated for five-level inverter. The scheme can be extended to higher-level inverters. Simulation results have been presented for four-level and five-level neutral point clamped inverters. The total harmonic distortion have been calculated and compared with lower levels. The results have been good agreement with the published work.

Keywords: Multi-level inverters, SVPWM, neutral point clamped inverter, Induction motor, THD.

1. Introduction

The recent industrial applications require high power apparatus. The classical Two-level inverters may not be useful in high voltage range (above 2 kV) due to limited rating of switches. In conventional two-level inverter configurations, the harmonic contents reduction of an inverter output current is achieved mainly by increasing the switching frequency. But the switching frequency is restricted by the switching losses in high power and high voltage applications [2]. In such applications, multilevel inverters have been widely used in recent years for the advantage of low harmonic output at low switching frequency. At the same time, low blocking voltage in the switching devices can be achieved. As a result, multilevel power inverter structure has been introduced as an alternative in high power and medium voltage situations [3]. There are mainly three types of multilevel inverter topologies: neutral point clamped inverter, cascaded inverter and flying capacitor inverter[4]. The multilevel inverters have several advantages like i) *Staircase waveform quality*: Multilevel inverters can generate the output voltages with very less distortion. As level of

inversion increases, the output voltage is nearer to sinusoidal. ii) *Voltage stress reduction*: Multilevel inverter can reduce voltage stress on switching devices. iii) *Input current quality*: Multilevel inverters can draw input current with less distortion. iv) *Reduction in Total Harmonic Distortion (THD)*: Multilevel inverter output can have less harmonic content compared to two level waveforms operating at the same switching frequency. The paper is organized as follows: Section 2 of the paper introduces the neutral point clamped multi-level inverters. Section 3 illustrates the space vector pulse width modulation for multi-level inverters and proposed algorithm used to determine the location of the reference voltage vector and calculation of switching on-times. Section 4 presents simulation results and discussions that demonstrate the viability and high performance of the proposed algorithm. Finally Section 5 concludes the paper.

2. Neutral Point Clamped Multilevel Inverters

In these inverters, the voltage across semiconductor switches are limited by diodes connected to various DC levels as such it is called Diode Clamped Multilevel inverters. According to the original invention, the concept can be extended to any number of levels by increasing the number of capacitors addition across source dc-bus. Early descriptions of this topology were limited to three-levels where two capacitors are connected across the dc bus resulting in one additional level. The additional level was the neutral point of the dc bus, so the terminology Neutral Point Clamped inverter was introduced. Because of industrial developments over the past several years, the three-level inverter is now used extensively in industry applications. Diode clamped converter is connected in series-parallel fashion to the three phase grid to compensate for distortion in utility voltage. The functional diagram of an n-level NPC converter is shown in figure 1.



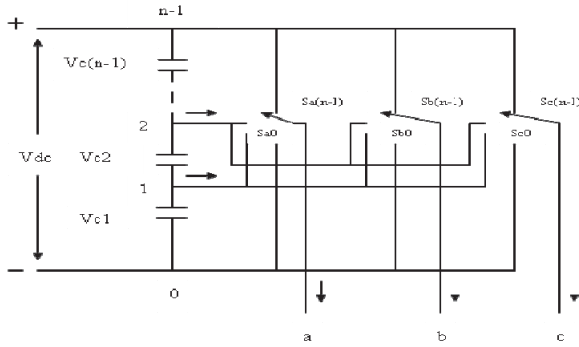


Figure 1. Functional diagram of n-level NPC Inverter

To result a defined output voltage, $(n-1)$ consecutive switches of each leg must be in the On-state. All possible combinations of the above functional diagram can be summarized by the equation (1).

$$\sum S_{ij} = 1 \quad \text{with} \quad i = \{a, b, c\} \quad (1)$$

The variables S_{ij} are the control functions of the single-pole n -throw switches. These variables define the position of switches, so that they have the unity value when the i output is connected to j point; other wise they are zero. Referring all of the voltages to the lower DC-link voltage level ("0" reference) each output voltage consists of contributions by a determined number of consecutive capacitors:

$$V_{i0} = \sum_{j=1}^{n-1} \left(S_{ij} \sum_{p=1}^j V_{c_p} \right) \quad \text{with} \quad i = \{a, b, c\} \quad (2)$$

When balanced distribution of DC-link voltage among the capacitors is assumed:

$$V_{i0} = \frac{V_{dc}}{n-1} \sum_{j=1}^{n-1} j S_{ij} \quad \text{with} \quad i = \{a, b, c\} \quad (3)$$

The voltage waveform obtained from n -level inverter is shown in figure 2.

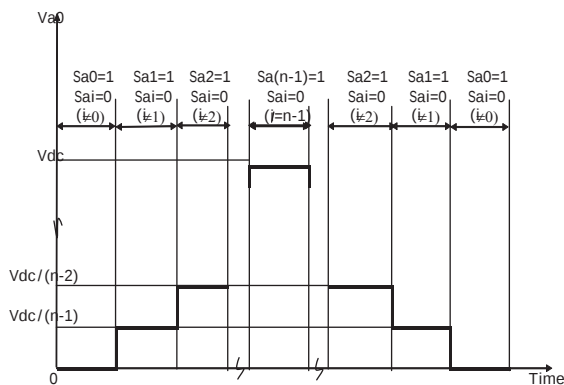


Figure 2. n-level inverter voltage waveform

Since its application in 1980s, three-level NPC inverter is most practical and widely studied multilevel inverter topology. One application of the multilevel diode-clamped inverter is an interface between a high-voltage dc transmission line and an ac transmission line. Another application would be as a variable speed drive for high-power medium-voltage (2.4 kV to 13.8 kV) motor.

The advantages of NPC inverter are:

- (i) All of the phases share a common dc bus, which minimizes the capacitance requirements of the inverter. For this reason, a back-to-back topology is not only possible but also practical for uses such as a high-voltage back-to-back inter-connection or an adjustable speed drive.
- (ii) The capacitors can be pre-charged as a group.
- (iii) Efficiency is high for fundamental frequency switching.

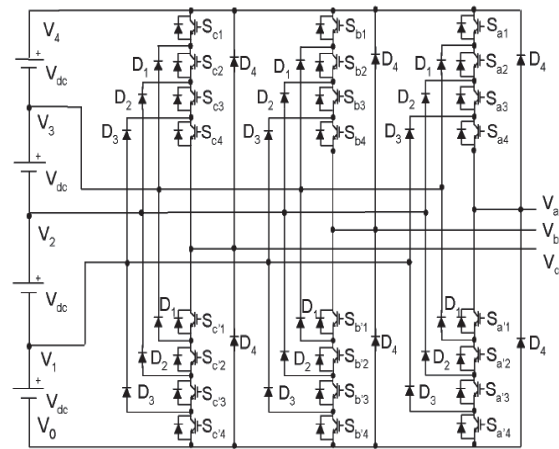


Figure 3. Five-level NPC inverter topology

In case of five-level inverters four switches from each phase-leg will be ON at any point time to produce predetermined output at phases. The possible switching combinations will be 125 with 64 redundant states.

Table -I. General characteristics of Multi-level inverters

NPC Inverter	a	b	c	d	e	f	g
n -level	$6(n-1)$	$(n-1)$	n^3	$n^3 - (n-1)^3$	$(n-1)^3$	$2n-1$	$4n-3$
2-level	6	1	8	7	1	3	5
3-level	12	2	27	19	8	5	9
4-level	18	3	64	37	27	7	13
5-level	24	4	125	61	64	9	17

a=number of switches with free wheeling diodes

b=number of consecutive switches of each leg to be in ON-state

c=number of different voltage states of the inverter

d=number of unique voltage states of the inverter

e=number of redundant voltage states of the inverter

g=phase voltage levels



3. Space Vector Pulse Width Modulation for Multi-level Inverters

Various Pulse Width Modulation (PWM) algorithms have been studied to control the multilevel inverter systems and Space Vector Modulation (SVPWM) method is a valid one. The most significant advantages of SVPWM are fast dynamic response and wide linear range of fundamental voltage compared with the conventional PWM. But when it is applied to the diode clamped inverter and flying capacitor inverter, the SVPWM strategy also has to solve the neutral-point voltage unbalance problem [5][6]. There are three main steps to obtain the proper switching states during each sampling period for the SVPWM method [7]:

- 1) Choose the proper basic vectors.
- 2) Calculate the dwelling time of each selected vectors.
- 3) Select the proper sequence of the pulse

It is not easy to implement the steps above directly with the reference voltage vector amplitude and phase, especially in case of high-level inverters. The most direct way of calculating the time to decompose all the vectors into real part and imaginary part. Another efficient way is to calculate the time according to reference voltage vector of each phase [7] [8]. Many papers discussed the methods to solve the problem and they mainly focused on last two processes because the first one is fairly simple for three level inverters. But for higher level inverters, all the three processes become complex. In this paper a qualitative SVPWM algorithm is proposed to implement the above process for high-level inverters.

The space vector (V_s) constituted by the pole voltages of inverter V_{az} , V_{bz} and V_{cz} with 120° phase displacement can be defined as:

$$V_s = V_{az} + V_{bz} \exp[j(2\pi/3)] + V_{cz} \exp[j(4\pi/3)] \quad (4)$$

The space vector, V_s can be resolved into two rectangular components namely V_α and V_β as indicated below:

$$V_s = V_\alpha + jV_\beta \quad (5)$$

$$\therefore \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} \quad (6)$$

Equating volt-seconds along α axis gives:

$$V_s \cos \alpha \times T_s = V_{dc} \times T_1 + (V_{dc} \cos 60^\circ) \times T_2 \quad (7)$$

Equating volt-seconds along β axis gives:

$$V_s \sin \alpha \times T_s = (V_{dc} \sin 60^\circ) \times T_2 \quad (8)$$

Solving equation (7) and equation (8) gives the values for the on- time periods T_1 , T_2 and T_0 are

$$T_1 = \frac{V_s \times T_s \times \sin(\pi/3 - \alpha)}{V_{dc} \times \sin(\pi/3)} \quad (9)$$

$$T_2 = \frac{V_s \times T_s \times \sin \alpha}{V_{dc} \times \sin(\pi/3)} \quad (10)$$

$$T_0 = T_s - (T_1 + T_2) \quad (11)$$

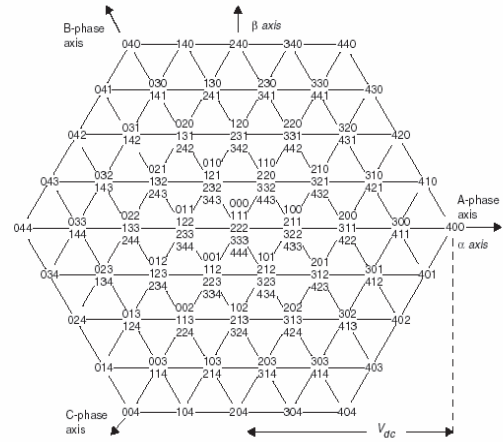


Figure 4. Space Vector diagram of Five-level inverter

In Multilevel inverters the reference voltage vector can be reproduced in the average sense by switching amongst the inverter states situated at the vertices, which are in closest proximity to it. The duty cycles (ON time for each state) will be found by equating volt-seconds of reference voltage with nearest three states.

$$m = d_1 V_1 + d_2 V_2 + d_3 V_3 \quad (12)$$

Where d_1 , d_2 and d_3 are duty cycles of voltage vectors (states) (V_1 , V_2 and V_3) in inverter SV diagram nearer to the reference vector and m is the voltage reference vector.

Calculation of duty cycles:

The vector states at vertices of each region can be identified from space vector diagrams shown in figure 4. Consider the space vector diagram of sector 1 of 5-level inverter, shown in figure 5. The reference vector m_3 is located in region 2 of 3 level inverter. m_4 and m_5 are reference vectors located in region 8 of four-level inverter and region 12 in five-level inverter, respectively. m_{x1} and m_{x2} ($x=3$ or 4 or 5) are projections of reference vectors on to zero and sixty degrees axes (angle ' θ ' is angle made by reference vector from zero axes i.e., starting of sector 1; be noted m_3 , m_4 and m_5 are having different angle ' θ ' value). The reference vector can be synthesized by sequential switching operation of nearest three switching states (vertices of the region in which reference vector is located).

The lengths of new vectors can be calculated as

$$m_1 = m \times (\cos \theta - \sin \theta / \sqrt{3})$$

$$m_2 = 2 \times m \times \sin \theta / \sqrt{3} \quad (13)$$

The values of m_1 and m_2 for reference vector in each region can be calculated with equation (13). The duty cycles of vertices of reference voltage will be m_1 , m_2 and



$[1-(m_1+m_2)]$. For example, with reference to m_3 (reference vector in region 2), the reference vector can be synthesized by switching vectors V_1 , V_2 and V_3 . It shall be important to note that duty cycle for switching state V_1 shall be length of the vector joining V_3 and V_1 , whereas, m_1 is the projection of reference vector m_3 from origin. As such, the corrected duty cycle for switching state V_1 in present case would be $(m_1-0.25)$. The length of vector joining V_3 and V_2 is m_2 . As such, corrected duty cycles for switching states V_1 , V_2 and V_3 would be $(m_1-0.25)$, m_2 and $(0.75-m_1-m_2)$ respectively.

The values of m_1 and m_2 are useful in identifying the region where reference vector is located, which is the major problem in multilevel inverters. The conditions for identifying reference vector location in each region and the corrected duty cycles for each of the level of inverter are shown in Table-II. Once the region is identified, the appropriate switching sequence of a region can be identified. The ON time period for each state can be calculated with duty cycles obtained:

$$T_{ON} \text{ for state1} = T_s \times m_1$$

$$T_{ON} \text{ for state2} = T_s \times m_2$$

$$T_{ON} \text{ for state3} = T_s \times [1 - (m_1+m_2)] \quad (14)$$

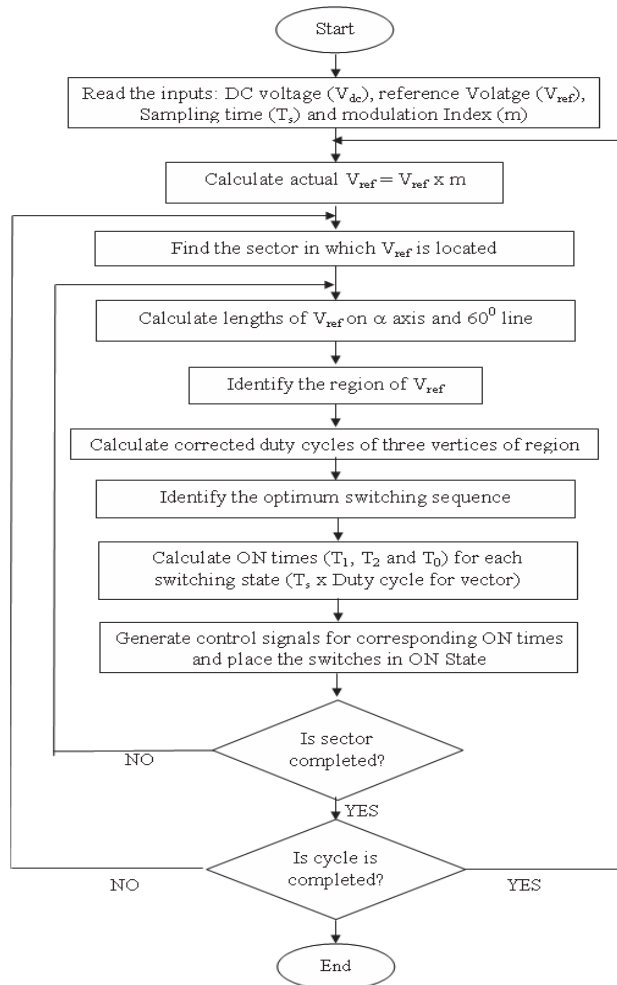


Figure 5. Flowchart of the SVPWM for multi-level inverters

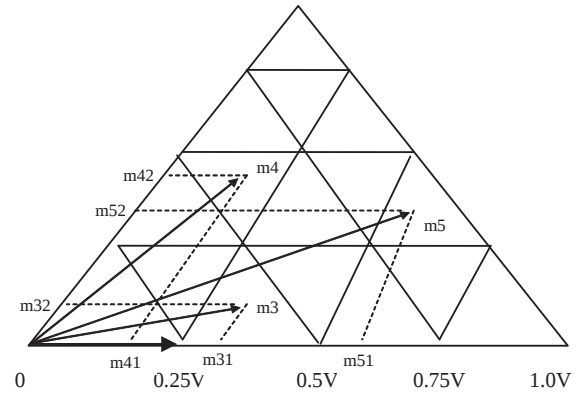


Figure 6. Sector 1 of five-level inverter

Table-II. Location of reference vector in Five-level inverter

Region	Condition for location of reference vector	Corrected m_1 , m_2 and m_3 for switching states
10	$0.75 < m_1 < 1$; $m_2 < 0.25$; $(1 + m_2) < 1$	$m_1 = m_1 - 0.75$; $m_2 = m_2$; $m_3 = 1 - m_1 - m_2$;
11	$0.5 < m_1 < 0.75$; $m_2 < 0.25$; $(m_1 + m_2) > 0.75$	$m_1 = 0.75 - m_1$; $m_2 = 0.25 - m_2$; $m_3 = m_1 + m_2 - 0.75$
12	$0.5 < m_1 < 0.75$; $0.25 < m_2 < 0.5$; $(m_1 + m_2) < 1$	$m_1 = m_1 - 0.5$; $m_2 = m_2 - 0.25$; $m_3 = 1 - m_1 - m_2$;
13	$0.25 < m_1 < 0.5$; $0.25 < m_2 < 0.5$; $(m_1 + m_2) > 0.75$	$m_1 = 0.5 - m_1$; $m_2 = 0.5 - m_2$; $m_3 = m_1 + m_2 - 0.75$;
14	$0.25 < m_1 < 0.5$; $0.5 < m_2 < 0.75$; $(m_1 + m_2) < 1$	$m_1 = m_1 - 0.25$; $m_2 = m_2 - 0.5$; $m_3 = 1 - m_1 - m_2$;
15	$m_1 < 0.25$; $0.5 < m_2 < 0.75$; $(m_1 + m_2) > 0.75$	$m_1 = 0.25 - m_1$; $m_2 = 0.75 - m_2$; $m_3 = m_1 + m_2 - 0.75$
16	$m_1 < 0.25$; $0.75 < m_2 < 1$; $(m_1 + m_2) < 1$	$m_1 = m_1$; $m_2 = m_2 - 0.75$; $m_3 = 1 - m_1 - m_2$;

Table-III. Switching sequence for Five-level inverter

Sector	Region	ON Sequence			
1	1	222	322	332	333
	2	322	422	432	433
	3	322	332	432	433
	4	332	432	442	443
	5	311	411	421	422
	6	311	321	421	422
	7	321	421	431	432
	8	321	331	431	432
	9	331	431	441	442
	10	300	400	410	411



Sector	Region	ON Sequence			
	11	300	310	410	411
	12	310	410	420	421
	13	310	320	420	421
	14	320	420	430	431
	15	320	330	430	431
	16	330	430	440	441
2	17	333	343	443	444
	18	332	342	442	443
	19	332	342	343	443
	20	232	242	342	343
	21	331	341	441	442
	22	331	341	342	442
	23	231	241	341	342
	24	231	241	242	342
	25	131	141	241	242
	26	330	340	440	441
	27	330	340	341	441
	28	230	240	340	341
	29	230	240	241	341
	30	130	140	240	241
	31	130	140	141	241
	32	030	040	140	141
3	33	000	010	011	111
	34	010	020	021	121
	35	010	011	021	121
	36	011	021	022	122
	37	131	141	142	242
	38	132	142	242	243
	39	132	142	143	243
	40	133	143	243	244
	41	133	143	144	244
	42	030	040	041	141
	43	031	041	141	142
	44	031	041	042	142
	45	032	042	142	143
	46	032	042	043	143
	47	033	043	143	144
	48	033	043	044	144
4	49	333	334	344	444
	50	233	234	244	344
	51	223	123	122	112
	52	223	224	234	334
	53	133	134	144	244
	54	123	133	144	244
	55	123	124	134	234
	56	113	123	124	224
	57	113	114	124	224
	58	033	034	044	144
	59	023	033	034	134
	60	023	024	034	134
	61	013	023	024	124
	62	013	014	024	124
	63	003	013	014	114
	64	003	004	014	114

Sector	Region	ON Sequence			
5	65	000	001	101	111
	66	001	002	102	112
	67	001	101	102	112
	68	101	102	202	212
	69	103	113	114	214
	70	102	103	113	213
	71	102	103	203	213
	72	202	203	213	313
	73	202	203	303	313
	74	003	004	104	114
	75	003	103	104	114
	76	103	104	204	214
	77	103	203	204	214
	78	203	204	304	314
	79	203	303	304	314
	80	303	304	404	414
6	81	333	433	434	444
	82	323	423	424	434
	83	323	423	433	434
	84	322	422	423	433
	85	302	303	313	413
	86	302	312	313	413
	87	312	412	413	423
	88	312	412	422	423
	89	311	411	412	312
	90	303	403	404	414
	91	303	403	413	414
	92	302	402	403	413
	93	302	402	412	413
	94	301	401	402	412
	95	301	401	411	412
	96	300	400	401	411

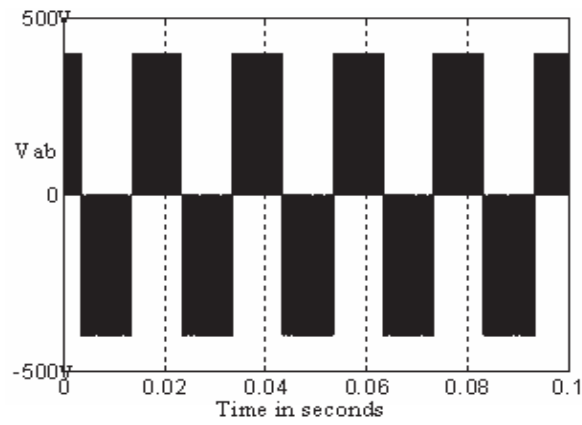
4. Simulation Results and Discussion

The simulation is carried out for the Two-level, Three-level, Four-level and Five-level inverters using a qualitative space vector PWM algorithm and the simulation results are presented. Figure 7 shows the line to line voltages of multi-level inverters. It shows the improvement of output voltage with the increase of level of inverter. The total harmonic distortion is shown in figure 8 and change in THD with level of inverter is shown in figure 10.. The stator current, speed and torque of induction motor are shown in figure 9.

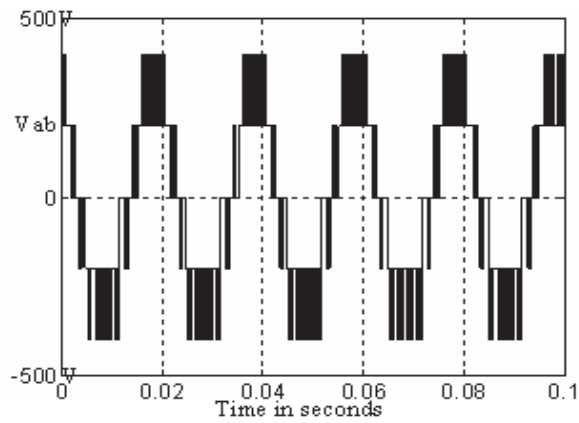
Simulation parameters are as follows:

SVPWM parameters: Modulation index $m = 0.8$;
 DC Supply voltage = 400V;
 No. of switching intervals = 192;
 Sampling time $T_s = 1.0417 \times 10^{-4}$ sec.
 Induction Motor: 2 HP, 1500W, 400V, 50Hz,
 4pole, 1430rpm, $R_s = 1.405 \Omega$;
 $R_r = 1.395 \Omega$; $L_s = 5.839 \text{ mH}$;
 $L_r = 5.839 \text{ mH}$; $L_m = 0.143 \text{ H}$;
 $J = 0.0131 \text{ Kg-m}^2$;
 $f_r = 0.002985 \text{ N-m-s}$

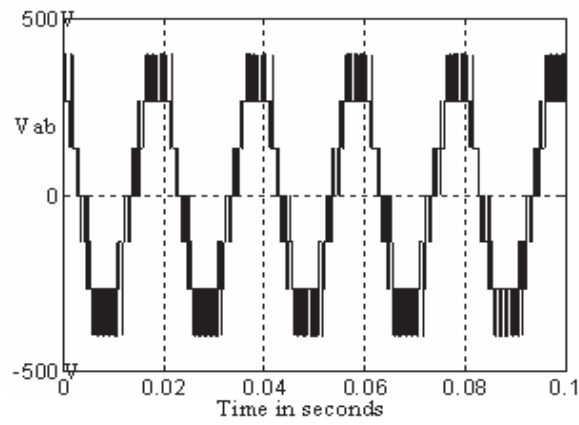




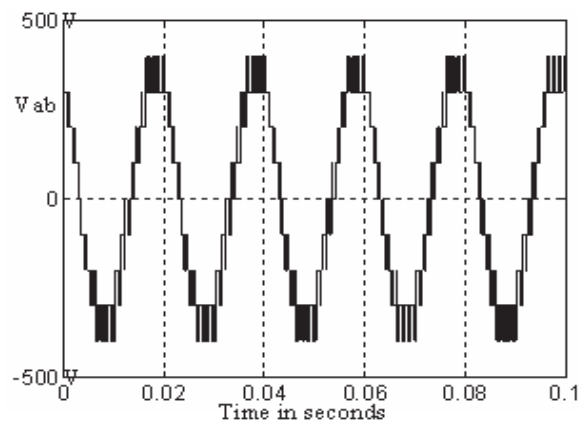
(a) two-level



(b) three-level

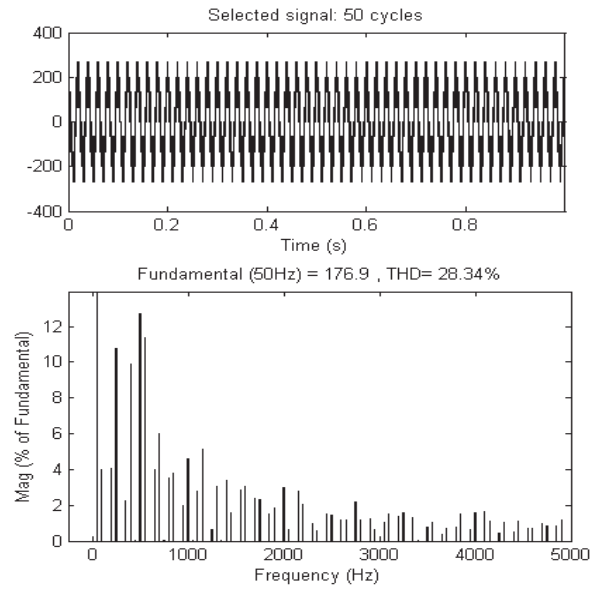


(c) four-level

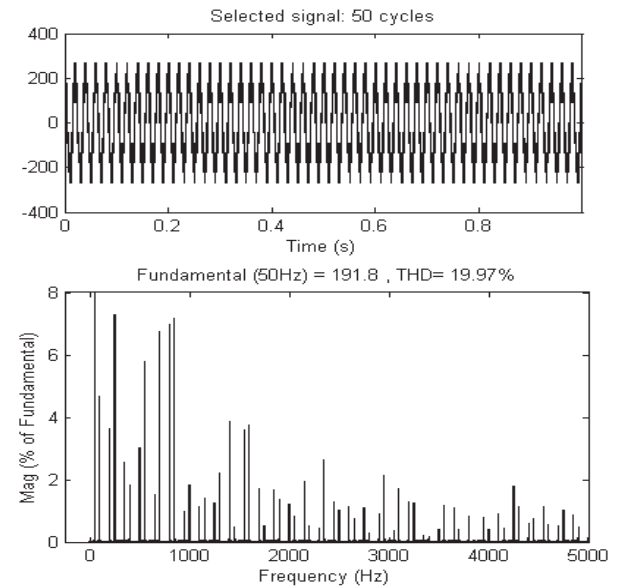


(d) five-level

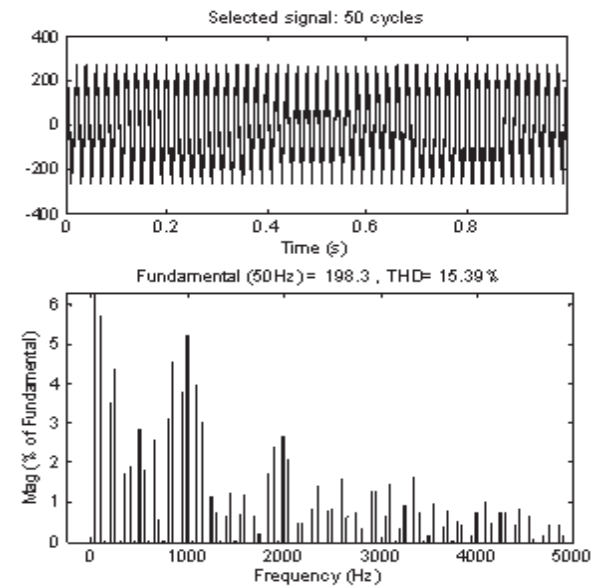
Figure 7. Line-Line voltages of Multi-level inverters.



(a) three-level



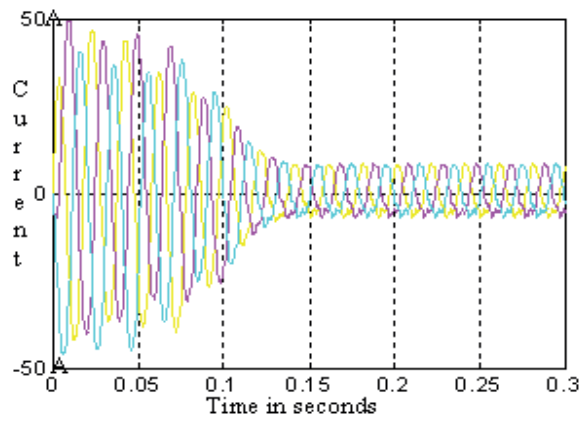
(b) four-level



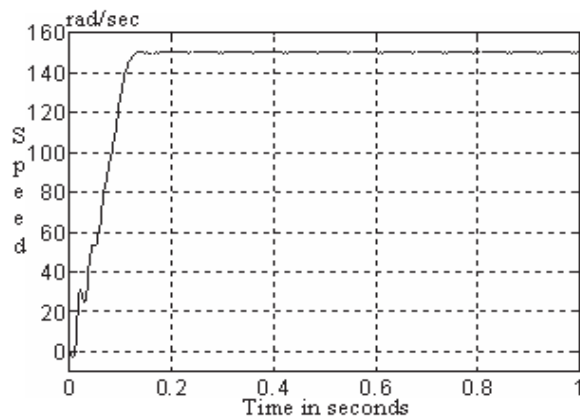
(c) five-level

Figure 8. FFT analysis and THD

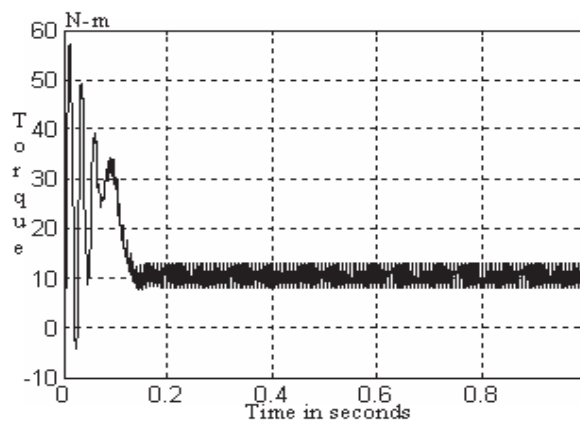




(a) stator currents



(b) motor speed



(c) Torque developed by motor

Figure 9. Stator current, Speed and Torque of Induction motor

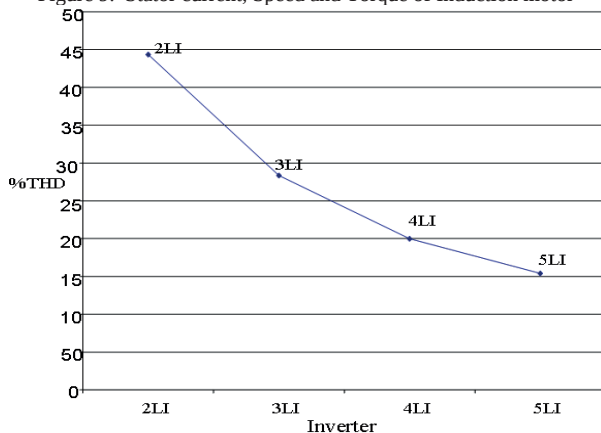


Figure 10. Inverter level Vs THD.

Table –IV. Simulation results with Full load on Induction motor

Parameter	Inverter			
	2LI	3LI	4LI	5LI
No. of spikes / Half cycle in I – steady state	20	6	6	5
Peak to Peak I - starting	88	79	85	84
Peak to Peak I – steady state	19	17	16	15
Time taken to reach I – steady state	0.16	0.18	0.16	0.14
Peak to Peak T - starting	62	54	59	61
T – steady state (mean)	10.20	10.44	10.3	10.45
Speed ripple	0.3	0.2	0.25	0.5
%THD	44.33	28.34	19.97	15.39

5. Conclusions

In this paper a qualitative space vector pulse width modulation algorithm has been described and applied to three-level, four-level and five-level inverters. Compared with conventional methods, this method has the advantage of ease implementing, especially for the inverters with more levels. From simulation results it is observed that the generated voltage spectrum is very much improved with increase the level of inverter. The total harmonic distortion (THD) is highly reduced as the level of the inverter is increases. The input current drawn by the induction motor is less distorted as level of inverter increases. The results have been presented and analysed in this paper.

6. References

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